

# Log-linear-complexity GLRT-optimal Noncoherent Sequence Detection for Orthogonal and RFID-oriented Modulations

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**Abstract**—Orthogonal modulation, for example, frequency-shift keying (FSK) or pulse-position modulation (PPM), is primarily used in relatively-low-rate communication systems that operate in the power-limited regime. Optimal noncoherent detection of orthogonally modulated signals takes the form of sequence detection and has exponential (in the sequence length) complexity when implemented through an exhaustive search among all possible sequences. In this work, for the first time in the literature, we present an algorithm that performs generalized-likelihood-ratio-test (GLRT) optimal noncoherent sequence detection of orthogonally modulated signals in flat fading with log-linear (in the sequence length) complexity. Moreover, for Rayleigh fading channels, the proposed algorithm is equivalent to the maximum-likelihood (ML) noncoherent sequence detector. Simulation studies indicate that the optimal noncoherent FSK detector attains coherent-detection performance when the sequence length is on the order of 100, offering a 3–5dB gain over the typical energy (single-symbol) detector. While the conventional exhaustive-search approach becomes infeasible for such sequence lengths, the proposed implementation requires a log-linear only number of operations, opening new avenues for practical deployments. Finally, we show that our algorithm also solves efficiently the optimal noncoherent sequence detection problem in contemporary radio frequency identification (RFID) systems.

**Index Terms**—Algorithm design and analysis, combinatorial mathematics, fading channels, FM0 coding, frequency-shift keying, generalized likelihood-ratio test, maximum-likelihood detection, noncoherent communication, pulse-position modulation, radio-frequency identification, sequence detection, wireless communication.

## I. INTRODUCTION

Orthogonal modulation is preferred to linear modulation (e.g., phase-shift keying (PSK), pulse-amplitude-modulation (PAM), quadrature amplitude modulation (QAM)) in relatively-low-rate communication systems that operate in the power-limited regime. A typical example is FSK which is primarily used (or considered for future use) in underwater communications [1]–[7], acoustic short-range communica-

tions [8], [9], power-line communications [10], [11], backscatter sensor networks and RFID [12]–[16], low-power wireless sensor networks [17], and cooperative communications [18]–[23]. To avoid the need for channel estimation (that induces added complexity at the receiver end and rate loss due to the necessary use of a pilot sequence), systems that utilize orthogonal modulation usually operate in the noncoherent mode; the receiver performs noncoherent (or blind) detection without any channel knowledge [8], [10], [13], [18]–[21], [23]–[27]. This is partly due to the simplicity of the single-symbol noncoherent detector which, for orthogonal modulation (e.g., FSK or PPM), is a simple energy detector [8], [18]–[21], [25]–[28].

However, due to channel-induced memory, the optimal noncoherent detector is no longer a single-symbol one but requires processing of the entire received sequence to make a decision on the entire data sequence, i.e., it is a sequence detector. In fact, noncoherent sequence detection may offer significant performance gains in comparison with conventional single-symbol noncoherent detection [29], [30]. This observation was first made in [31]–[34], in the context of  $M$ -ary PSK ( $M$ PSK), where it was shown that ML noncoherent sequence detection minimizes the sequence error probability, offering significant error rate performance gains over the conventional symbol-by-symbol noncoherent detection and attaining nearly-coherent detection performance for sufficiently long sequences. This is partly due to the fact that sequence detection exploits the correlation of the received symbols in the entire sequence (due to channel-induced memory), whereas symbol-by-symbol detection does not.

Regarding FSK modulation, noncoherent sequence detection has been considered in [10], [14], [23], [24], [35], [36]. Nevertheless, optimal sequence detection comes at a high price when implemented through an exhaustive search among all possible transmitted data sequences; its complexity is exponential in the sequence length [23], [35], [36]. In the context of power-line communications, to reduce the overall complexity, the authors in [10] propose a low-complexity suboptimal noncoherent FSK sequence detector. Work in [14], [16], in the context of scatter radio sensor networks, offered soft-decision metrics for noncoherent binary FSK sequence detection; however, each possible sequence belonged to a specific short block-length error-correcting (channel) code.

In this work, for the first time in the literature, we present an algorithm that performs GLRT-optimal noncoherent sequence detection of orthogonally modulated signals in flat fading

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with log-linear (in the sequence length) complexity. GLRT is independent of the fading channel distribution and, hence, is a practical option when the channel statistics are not known at the receiver. Moreover, we show that, for Rayleigh fading channels, the proposed algorithm is equivalent to the ML noncoherent sequence detector. Our algorithm utilizes principles that have been used for polynomial-complexity optimization in [37]–[40] and complements efficient optimal noncoherent detection techniques that have been developed for PSK [37], [38],<sup>1</sup> [42] and PAM or QAM [40], [43]–[45] signals.<sup>2</sup> We show that, as the sequence length increases, the proposed noncoherent scheme for orthogonally modulated signals attains nearly coherent performance, offering a 3–5dB gain over the typical energy (single-symbol) detector [8], [18]–[21], [25]–[28], whereas it does not require any channel knowledge. In contrast to the conventional exhaustive-search approach that requires an exponential number of operations, the proposed implementation of optimal<sup>3</sup> noncoherent detection requires a log-linear only (in the sequence length) number of operations, opening new avenues for practical deployments.

Moreover, we consider noncoherent sequence detection of FM0 signals. We recall that FM0 is a line-coding technique that is utilized by the current RFID standards [46]–[64]. For FM0, the optimal coherent detector operates on two consecutive received samples to make a decision for a single bit [46]. However, for noncoherent detection over channels whose coherence time spans more than two information symbols, such two-sample correlation is inadequate to allow optimal detection. In this work, we show that noncoherent sequence detection of zero-offset FM0 is equivalent to noncoherent sequence detection of uncoded binary FSK (BFSK). Hence, the proposed algorithm for orthogonally modulated signals also solves efficiently the optimal noncoherent detection problem in contemporary RFID systems. Finally, we show that noncoherent sequence detection of antipodal FM0 signals is equivalent to noncoherent sequence detection of uncoded binary PSK, allowing the use of relevant log-linear-complexity optimal sequence detectors [37], [38], [41], [42].

The rest of this paper is organized as follows. Section II presents the signal model for orthogonal modulation and the corresponding ML and GLRT noncoherent sequence detection algorithms. Sections III and IV describe how the new algorithms of Section II can be utilized for optimal noncoherent detection in contemporary RFID systems. In Section V, we study the performance of the proposed schemes. Finally, a few conclusions are drawn in Section VI.

Notation: Nonbold lower-case letters (e.g.,  $x$ ) will stand for variables. Vectors and matrices will be denoted by lower-case (e.g.,  $\mathbf{x}$ ) and capital (e.g.,  $\mathbf{A}$ ), respectively, bold characters. Symbols  $(\cdot)^T$  and  $(\cdot)^H$  will denote the transpose and hermitian, respectively, of a vector or matrix. Real-part operation is denoted by  $\Re\{\cdot\}$ . The proper complex Gaussian distribution with mean  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$  is denoted

by  $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ . Finally,  $\|\mathbf{x}\|$  stands for the euclidean norm of vector  $\mathbf{x}$ .

## II. ORTHOGONAL MODULATION

In this section, we present a novel algorithm that performs optimal noncoherent sequence detection of orthogonally modulated signals in flat fading with log-linear complexity. We note that we consider a block flat-fading channel and our proposed algorithm performs GLRT-optimal detection under a Gaussian assumption about the noise. Moreover, under an added Rayleigh assumption about the channel, our algorithm is also the ML detector.

Although our developments hold for any orthogonally modulated signaling technique, we choose to present them in the context of FSK.

### A. Signal Model and Optimal Noncoherent Detection

$M$ -ary FSK (MFSK) utilizes  $M$  sub-carrier frequencies to modulate the information symbol  $x \in \mathcal{M} \triangleq \{1, 2, \dots, M\}$ . For a single-symbol period, the transmitted MFSK waveform is given by  $u_x(t)$  which is selected from the available set of  $M$  waveforms defined by [28]

$$u_m(t) = \sqrt{\frac{P}{T}} e^{j2\pi f_m t}, \quad 0 \leq t < T, \quad m = 1, 2, \dots, M. \quad (1)$$

In (1),  $P$  and  $T$  denote signal strength and nominal duration, respectively, and  $f_m, m = 1, 2, \dots, M$ , are the utilized frequencies which, in noncoherent FSK, must satisfy the orthogonality condition, i.e.,  $|f_m - f_{m'}| = k\frac{1}{T}$ , for some  $k \in \mathbb{N}$ ,  $\forall m, m' \in \mathcal{M}$  with  $m \neq m'$ . If the modulated waveform is transmitted through a flat-fading channel [10], [23], [26], [35], [36], [65]–[67], then the received signal, after downconversion, is written as

$$r(t) = hu_x(t) + n(t) \quad (2)$$

where  $h$  is a complex number that models signal attenuation and phase change due to the channel<sup>4</sup> and  $n(t)$  is a zero-mean complex Gaussian process with variance  $\sigma_w^2$ , modeling thermal noise at the receiver. The optimal receiver correlates the received signal  $r(t)$  with all  $M$  signaling waveforms  $u_1(t), u_2(t), \dots, u_M(t)$  to produce samples

$$r_m = \frac{1}{\sqrt{P}} \int_0^T r(t)u_m^*(t)dt, \quad m = 1, 2, \dots, M. \quad (3)$$

If the orthogonality condition is satisfied, then

$$r_m = \begin{cases} \sqrt{P}h + n_m, & m = x, \\ n_m, & m \neq x, \end{cases} \quad (4)$$

where  $n_1, n_2, \dots, n_M$  are independent zero-mean circularly symmetric complex Gaussian variables with variance  $\sigma_w^2$  [28].

<sup>1</sup>The algorithm of [37], [38] reappeared in [41].

<sup>2</sup>The method in [40] applies to QAM constellations with independent in-phase and quadrature components.

<sup>3</sup>Throughout the paper, when we use the term ‘‘optimal sequence detection,’’ we refer to GLRT-optimal sequence detection.

<sup>4</sup>We assume that the channel fading coefficient is the same over each sub-carrier frequency.

Consequently, for a single-symbol duration, the received vector becomes

$$\underbrace{\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix}}_{\mathbf{r}} = \sqrt{P}h\mathbf{e}_x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}}_{\mathbf{n}} \quad (5)$$

where  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_M)$  and

$$\mathbf{e}_x = \underbrace{[0 \dots 0]_{x-1}}_{x-1} 1 \underbrace{[0 \dots 0]_{M-x}}_{M-x}^T \quad (6)$$

is the  $x$ th column,  $x = 1, 2, \dots, M$ , of the  $M \times M$  identity matrix  $\mathbf{I}_M$ . For notation simplicity, we also define the set

$$\mathcal{I}_M \triangleq \{\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_M\} \quad (7)$$

that consists of the  $M$  columns of  $\mathbf{I}_M$ .

If the actual channel realization is not available to the receiver and is modeled as a random variable, then this variable appears in consecutive received vectors implying that consecutive received vectors are no longer independent (even under the condition of known transmitted symbols). In other words, the channel induces memory in the received signal; as a result, optimal detection requires processing of a sequence of received vectors.

Let  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_N]^T \in \mathcal{M}^N$  be the transmitted  $N \times 1$  information-symbol sequence. If  $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N$  are the corresponding received vectors (per information symbol) given by (5), then we may form the received vector for the entire sequence  $\mathbf{x}$  as

$$\mathbf{y} \triangleq \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \sqrt{P}h \underbrace{\begin{bmatrix} \mathbf{e}_{x_1} \\ \mathbf{e}_{x_2} \\ \vdots \\ \mathbf{e}_{x_N} \end{bmatrix}}_{\mathbf{s}} + \mathbf{w} \quad (8)$$

where  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{MN})$ ,  $\mathbf{s}$  is the column-wise concatenation of the  $N$  transmitted signal vectors  $\mathbf{e}_{x_1}, \mathbf{e}_{x_2}, \dots, \mathbf{e}_{x_N}$ , and  $\mathbf{y}$  is the column-wise concatenation of the  $N$  received signal vectors  $\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N$ . Both  $\mathbf{s}$  and  $\mathbf{y}$  have size  $MN \times 1$ .

The ML noncoherent detector maximizes the conditional probability density function (pdf) of  $\mathbf{y}$  given  $\mathbf{s}$ , that is, the optimal decision is given by

$$\hat{\mathbf{s}}^{\text{ML}} = \arg \max_{\mathbf{s} \in \mathcal{I}_M^N} f(\mathbf{y}|\mathbf{s}) \quad (9)$$

where  $f(\cdot|\cdot)$  stands for the conditional pdf of the observation vector given the transmitted symbol sequence and

$$\mathcal{I}_M^N = \left\{ \begin{bmatrix} \mathbf{e}_{x_1} \\ \mathbf{e}_{x_2} \\ \vdots \\ \mathbf{e}_{x_N} \end{bmatrix} : \mathbf{e}_{x_i} \in \mathcal{I}_M, x_i \in \mathcal{M}, i = 1, 2, \dots, N \right\}. \quad (10)$$

Note that  $\|\mathbf{s}\|^2 = N$ , for any  $\mathbf{s} \in \mathcal{I}_M^N$ . It can be shown that, for Rayleigh fading (i.e.,  $h \sim \mathcal{CN}(0, \sigma_h^2)$ ) [26], [47], [52], [58], [65], [67], the received symbol vector  $\mathbf{y}$  given the transmitted

sequence  $\mathbf{s}$  follows a proper complex Gaussian distribution with mean  $\mathbb{E}[\mathbf{y}|\mathbf{s}] = \mathbf{0}$  and covariance matrix

$$\mathbf{C}_{\mathbf{y}|\mathbf{s}} \triangleq \mathbb{E}[\mathbf{y}\mathbf{y}^H|\mathbf{s}] = P\sigma_h^2\mathbf{s}\mathbf{s}^T + \sigma_w^2\mathbf{I}_{MN}. \quad (11)$$

Consequently, the ML optimization problem in (9) can be rewritten as

$$\hat{\mathbf{s}}^{\text{ML}} = \arg \max_{\mathbf{s} \in \mathcal{I}_M^N} \frac{1}{|\mathbf{C}_{\mathbf{y}|\mathbf{s}}|} e^{-\mathbf{y}^H \mathbf{C}_{\mathbf{y}|\mathbf{s}}^{-1} \mathbf{y}}. \quad (12)$$

Exploiting identities for the determinant and inverse of a rank-1 update [68] and the fact that  $\|\mathbf{s}\|^2 = N$ , we obtain

$$\begin{aligned} |\mathbf{C}_{\mathbf{y}|\mathbf{s}}| &= |\sigma_w^2 \mathbf{I}_{MN} + P\sigma_h^2 \mathbf{s}\mathbf{s}^T| = |\sigma_w^2 \mathbf{I}_{MN}| \left( 1 + \frac{P\sigma_h^2}{\sigma_w^2} \|\mathbf{s}\|^2 \right) \\ &= \sigma_w^{2MN-2} (\sigma_w^2 + P\sigma_h^2 \|\mathbf{s}\|^2) \\ &= \sigma_w^{2MN-2} (\sigma_w^2 + P\sigma_h^2 N) \end{aligned} \quad (13)$$

and

$$\begin{aligned} \mathbf{C}_{\mathbf{y}|\mathbf{s}}^{-1} &= (\sigma_w^2 \mathbf{I}_{MN} + P\sigma_h^2 \mathbf{s}\mathbf{s}^T)^{-1} \\ &= \frac{1}{\sigma_w^2} \mathbf{I}_{MN} - \frac{P\sigma_h^2}{\sigma_w^4 + P\sigma_h^2 \sigma_w^2 \|\mathbf{s}\|^2} \mathbf{s}\mathbf{s}^T \\ &= \frac{1}{\sigma_w^2} \mathbf{I}_{MN} - \frac{P\sigma_h^2}{\sigma_w^4 + P\sigma_h^2 \sigma_w^2 N} \mathbf{s}\mathbf{s}^T. \end{aligned} \quad (14)$$

If we substitute (13) and (14) in (12), we obtain

$$\begin{aligned} \hat{\mathbf{s}}^{\text{ML}} &= \arg \max_{\mathbf{s} \in \mathcal{I}_M^N} \left\{ -\mathbf{y}^H \left( \frac{1}{\sigma_w^2} \mathbf{I}_{MN} - \frac{P\sigma_h^2}{\sigma_w^4 + P\sigma_h^2 \sigma_w^2 N} \mathbf{s}\mathbf{s}^T \right) \mathbf{y} \right. \\ &\quad \left. - \ln (\sigma_w^{2MN-2} (\sigma_w^2 + P\sigma_h^2 N)) \right\} \\ &= \arg \max_{\mathbf{s} \in \mathcal{I}_M^N} |\mathbf{s}^T \mathbf{y}|. \end{aligned} \quad (15)$$

Substituting  $\mathbf{s} = [\mathbf{e}_{x_1}^T \ \mathbf{e}_{x_2}^T \ \dots \ \mathbf{e}_{x_N}^T]^T$  in (15), the ML rule is rewritten in terms of the information sequence  $\mathbf{x}$  as<sup>5</sup>

$$\hat{\mathbf{x}}^{\text{ML}} = \arg \max_{\mathbf{x} \in \mathcal{M}^N} |y_1[x_1] + y_2[x_2] + \dots + y_N[x_N]|. \quad (16)$$

The equivalent expressions (15) and (16) represent the optimal decision rule when the channel coefficient follows a Rayleigh distribution.

If, on the other hand, the channel distribution is not Rayleigh or is unknown, then we may consider joint channel estimation and data detection, i.e., GLRT sequence detection [44], according to which,

$$\begin{aligned} \hat{\mathbf{s}}^{\text{GLRT}} &= \arg \min_{\mathbf{s} \in \mathcal{I}_M^N} \left\{ \min_{h \in \mathbb{C}} \left\| \mathbf{y} - \sqrt{P}h\mathbf{s} \right\|^2 \right\} \\ &= \arg \min_{\mathbf{s} \in \mathcal{I}_M^N} \left\| \mathbf{y} - \frac{\mathbf{s}^T \mathbf{y}}{\|\mathbf{s}\|^2} \mathbf{s} \right\|^2 \\ &= \arg \max_{\mathbf{s} \in \mathcal{I}_M^N} \frac{|\mathbf{y}^H \mathbf{s}|^2}{\|\mathbf{s}\|^2} = \arg \max_{\mathbf{s} \in \mathcal{I}_M^N} |\mathbf{s}^T \mathbf{y}|. \end{aligned} \quad (17)$$

<sup>5</sup>We use the notation  $\mathbf{y}_n = \begin{bmatrix} y_n[1] \\ y_n[2] \\ \vdots \\ y_n[M] \end{bmatrix}$  to represent the  $M \times 1$  vector  $\mathbf{y}_n$ ,  $n = 1, 2, \dots, N$ .

Hence, the ML optimization problem in (15) and the GLRT optimization problem in (17) are equivalent. This equivalence between noncoherent ML and GLRT has been also demonstrated in the context of equal-energy signals and uniformly distributed over  $[0, 2\pi)$  channel phase [29], [43]. A straightforward approach to solve (15) and (17) (or, equivalently, (16)) would be an exhaustive search among all  $M^N$  sequences  $\mathbf{x} \in \mathcal{M}^N$ . However, such a solver would be impractical even for moderate values of  $N$ , since its complexity is on the order of  $\mathcal{O}(M^N)$ , i.e., it grows exponentially with  $N$ . In the following, we present an algorithm that solves the above problems with log-linear complexity  $\mathcal{O}(N \log N)$ . Similar algorithms have been developed for polynomial-time optimal noncoherent detection of PSK signals in [37], [38], [41], [42] and PAM or QAM signals in [40], [43]–[45].

### B. Log-linear-complexity Optimal Detection

First, we present the proposed algorithm for BFSK ( $M = 2$ ). Then, we generalize to any  $M \geq 2$ . In either case, we begin by rewriting the optimal detection rule in (16) as

$$\begin{aligned} & \max_{\mathbf{x} \in \mathcal{M}^N} |y_1[x_1] + y_2[x_2] + \dots + y_N[x_N]| \\ &= \max_{\mathbf{x} \in \mathcal{M}^N} \max_{\phi \in [0, 2\pi)} \Re \{ e^{-j\phi} (y_1[x_1] + y_2[x_2] + \dots + y_N[x_N]) \} \\ &= \max_{\phi \in [0, 2\pi)} \max_{\mathbf{x} \in \mathcal{M}^N} \left\{ \Re \{ e^{-j\phi} y_1[x_1] \} + \Re \{ e^{-j\phi} y_2[x_2] \} + \dots \right. \\ & \quad \left. \dots + \Re \{ e^{-j\phi} y_N[x_N] \} \right\}. \end{aligned} \quad (18)$$

1) *Optimal algorithm for  $M = 2$* : For a given point  $\phi \in [0, 2\pi)$ , the innermost maximization in (18) is separable for each  $x_n$  and, hence, splits into independent maximizations for any  $n = 1, 2, \dots, N$ , as

$$\begin{aligned} \hat{x}_n &= \arg \max_{x_n \in \{1, 2\}} \Re \{ e^{-j\phi} y_n[x_n] \} \\ &\Leftrightarrow \Re \{ e^{-j\phi} y_n[1] \} \underset{\hat{x}_n=2}{\overset{\hat{x}_n=1}{\geq}} \Re \{ e^{-j\phi} y_n[2] \} \\ &\Leftrightarrow \Re \{ e^{-j\phi} (y_n[1] - y_n[2]) \} \underset{\hat{x}_n=2}{\overset{\hat{x}_n=1}{\geq}} 0 \\ &\Leftrightarrow \cos(\phi - \underline{y_n[1] - y_n[2]}) \underset{\hat{x}_n=2}{\overset{\hat{x}_n=1}{\geq}} 0 \end{aligned} \quad (19)$$

where  $\hat{x}_n$  is the decision on the  $n$ th information symbol  $x_n$  of the sequence of  $N$  consecutive symbols and  $\underline{z}$  denotes the angle of the complex number  $z$ .

According to (18), to obtain the optimal sequence  $\hat{\mathbf{x}}^{\text{ML}}$  in (16), it suffices to let  $\phi$  vary from 0 to  $2\pi$  and, for each value of  $\phi$ , determine the corresponding sequence  $\hat{\mathbf{x}} = [\hat{x}_1 \hat{x}_2 \dots \hat{x}_N]^T$  using (19) for  $n = 1, 2, \dots, N$ . Then, one of the collected sequences that we meet as  $\phi$  varies from 0 to  $2\pi$  will be  $\hat{\mathbf{x}}^{\text{ML}}$ .

Interestingly, as  $\phi$  scans  $[0, 2\pi)$ , the decision  $\hat{x}_n$  changes, according to (19), only when

$$\begin{aligned} & \cos(\phi - \underline{y_n[1] - y_n[2]}) = 0 \\ \Leftrightarrow \phi &= \pm \underbrace{\frac{\pi}{2} + \underline{y_n[1] - y_n[2]}}_{\phi_n^{(1)}, \phi_n^{(2)}} \pmod{2\pi}. \end{aligned} \quad (20)$$

Hence, the sequence decision  $\hat{\mathbf{x}} = [\hat{x}_1 \hat{x}_2 \dots \hat{x}_N]^T$  changes only at  $\phi_1^{(1)}, \phi_1^{(2)}, \phi_2^{(1)}, \phi_2^{(2)}, \dots, \phi_N^{(1)}, \phi_N^{(2)}$ . In the following, we assume that the above  $2N$  points are distinct and nonzero, i.e.,  $\phi_n^{(j)} \neq \phi_l^{(k)}$  and  $\phi_n^{(j)} \neq 0$ , for any  $j, k \in \{1, 2\}$  and  $n, l \in \{1, 2, \dots, N\}$  with  $n \neq l$ . The case where the above assumption does not hold is examined separately in Footnote 6.

Since the  $2N$  points are distinct, only one element of the sequence changes at each such point. If we sort the above points in ascending order, i.e.,

$$(\theta_1, \theta_2, \dots, \theta_{2N}) = \text{sort} \left( \phi_1^{(1)}, \phi_1^{(2)}, \phi_2^{(1)}, \phi_2^{(2)}, \dots, \phi_N^{(1)}, \phi_N^{(2)} \right), \quad (21)$$

then the decision  $\hat{\mathbf{x}}$  remains constant in each one of the  $2N$  intervals

$$C_0 = (0, \theta_1), C_1 = (\theta_1, \theta_2), \dots, C_{2N-1} = (\theta_{2N-1}, \theta_{2N}). \quad (22)$$

Note that we ignore  $(\theta_{2N}, 2\pi)$  because it gives the same sequence  $\hat{\mathbf{x}}$  with  $C_0 = (0, \theta_1)$ .

Our objective is the identification of the  $2N$  sequences  $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{2N-1}$  (that correspond to the  $2N$  intervals  $C_0, C_1, \dots, C_{2N-1}$ ), one of which is  $\hat{\mathbf{x}}^{\text{ML}}$ . We begin by setting  $\phi = 0$  and determining the sequence  $\hat{\mathbf{x}}_0$  by (19) for  $n = 1, 2, \dots, N$ . Note that  $\hat{\mathbf{x}}_0$  corresponds to interval  $C_0$ . Then, we consider  $\phi = \theta_1$  and invert the decision symbol  $x_n$ ,  $n \in \{1, 2, \dots, N\}$ , which produced  $\theta_1$ . This way, we obtain the new sequence  $\hat{\mathbf{x}}_1$  that corresponds to  $C_1$ . We continue by considering  $\phi = \theta_2$  and repeating the above procedure to obtain  $\hat{\mathbf{x}}_2$  that corresponds to  $C_2$ . Subsequently, we set  $\phi = \theta_3$  to obtain  $\hat{\mathbf{x}}_3$  and continue similarly with  $\phi = \theta_4, \theta_5, \dots, \theta_{2N-1}$  to determine all  $2N$  sequences  $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{2N-1}$ . Then, we compare them against the metric of interest in (16) to identify the optimal one which is  $\hat{\mathbf{x}}^{\text{ML}}$ .

The proposed algorithm that we just described is presented in Fig. 1.<sup>6</sup> The overall complexity to produce the  $2N$  sequences  $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{2N-1}$  among which  $\hat{\mathbf{x}}^{\text{ML}}$  is dominated by the computational cost of the sorting operation in (21) which is on the order of  $\mathcal{O}(2N \log_2 2N) = \mathcal{O}(N \log N)$ .

**Improvements to the proposed algorithm:** We can simplify the algorithm that we described above by taking into account a few properties of the generated candidate sequences.

First, observe that the candidate sequence that we obtain at any  $\phi \in [0, \pi)$  is the complement of the candidate sequence

<sup>6</sup>If  $\phi_n^{(j)} = 0$  for some  $j \in \{1, 2\}$  and  $n \in \{1, 2, \dots, N\}$ , then  $\theta_1 = 0$ , implying that the candidate sequence  $\hat{\mathbf{x}}_0$  cannot be defined at  $\phi = 0$  due to sign ambiguity with respect to the symbol  $x_n$  that produced  $\theta_1 = 0$ . Then, we change the interval of  $\phi$  from  $[0, 2\pi)$  to  $[\theta^*, 2\pi + \theta^*)$ , where  $\theta^*$  is a point that belongs to the interval that precedes  $\theta_1 = 0$ . In this case, we select  $\theta^*$  to be the mid-point of the preceding interval  $(\theta_{2N}, 2\pi)$ , i.e.,  $\theta^* = \frac{\theta_{2N} - 2\pi}{2}$ , as shown at lines 6–10 of the algorithm in Fig. 1. Hence,  $\theta_{2N} < \theta^* \pmod{2\pi}$  and  $\theta^* < \theta_1 = 0$ , implying that the first two intervals in (22) change to  $C_0 = (\theta^*, \theta_1 = 0)$  and  $C_1 = (\theta_1 = 0, \theta_2)$  where the corresponding two candidate sequences  $\hat{\mathbf{x}}_0$  and  $\hat{\mathbf{x}}_1$  are uniquely defined.

Finally, if  $\phi_n^{(j)} = \phi_l^{(k)}$  for some  $j, k \in \{1, 2\}$  and  $n, l \in \{1, 2, \dots, N\}$  with  $n \neq l$ , then  $\theta_i = \theta_{i+1}$  for some  $i \in \{1, 2, \dots, 2N - 1\}$ , implying that interval  $C_i = (\theta_i, \theta_{i+1})$  does not exist and must be removed from the list of  $2N$  intervals in (22); the two adjacent intervals  $C_{i-1} = (\theta_{i-1}, \theta_i = \theta_{i+1})$  and  $C_{i+1} = (\theta_i = \theta_{i+1}, \theta_{i+2})$  that correspond to the candidate sequences  $\hat{\mathbf{x}}_{i-1}$  and  $\hat{\mathbf{x}}_{i+1}$  become successive and the total number of  $2N$  intervals (and sequences) is reduced by one. In this case, we let the algorithm produce the invalid intermediate candidate sequence  $\hat{\mathbf{x}}_i$  without increasing its complexity or affecting its optimality.

**Algorithm 1** Optimal Noncoherent Binary Orthogonal Detection

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**Input:**  $y_1, y_2, \dots, y_N$

- 1: **for**  $n = 1 : N$  **do**
- 2:  $\phi_n^{(1)} := +\frac{\pi}{2} + \frac{y_n[1] - y_n[2]}{2} \pmod{2\pi}$
- 3:  $\phi_n^{(2)} := -\frac{\pi}{2} + \frac{y_n[1] - y_n[2]}{2} \pmod{2\pi}$
- 4: **end for**
- 5:  $(\theta_1, \theta_2, \dots, \theta_{2N}) := \text{sort}(\phi_1^{(1)}, \phi_1^{(2)}, \phi_2^{(1)}, \phi_2^{(2)}, \dots, \phi_N^{(1)}, \phi_N^{(2)})$
- 6: **if**  $\theta_1 > 0$  **then**
- 7:  $\theta^* := 0$
- 8: **else**
- 9:  $\theta^* := \frac{\theta_{2N} - 2\pi}{2}$
- 10: **end if**
- 11: **for**  $n = 1 : N$  **do**
- 12:  $\hat{x}_n := \arg \max(\Re\{e^{-j\theta^*} y_n[1]\}, \Re\{e^{-j\theta^*} y_n[2]\})$
- 13: **end for**
- 14:  $\hat{\mathbf{x}}_0 := \hat{\mathbf{x}}$
- 15: **for**  $i = 1 : 2N - 1$  **do**
- 16: Invert in  $\mathbf{x}$  the symbol decision  $\hat{x}_n$  for which  $\theta_i$  was obtained
- 17:  $\hat{\mathbf{x}}_i := \hat{\mathbf{x}}$
- 18: **end for**
- 19:  $\hat{\mathbf{x}}^{\text{ML}} := \arg \max_{\mathbf{x} \in \{\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_{2N-1}\}} |y_1[x_1] + y_2[x_2] + \dots + y_N[x_N]|$

**Output:**  $\hat{\mathbf{x}}^{\text{ML}}$

---

Fig. 1. Optimal noncoherent sequence detection algorithm for binary orthogonal modulation and zero-offset FMO coding.

that we obtain at  $\phi + \pi$ .<sup>7</sup> Indeed, for the decision rule in (19), we observe that, for any  $\phi \in [0, \pi)$  and any  $n = 1, 2, \dots, N$ , we have

$$\cos(\phi - \frac{y_n[1] - y_n[2]}{2}) = -\cos((\phi + \pi) - \frac{y_n[1] - y_n[2]}{2}). \quad (23)$$

Hence, for any  $\phi \in [0, \pi)$ , (19) results in complementary sequences at  $\phi$  and  $\phi + \pi$ .

Then, from (20), we observe that  $|\phi_n^{(2)} - \phi_n^{(1)}| = \pi$ , hence  $\phi_n^{(1)}$  and  $\phi_n^{(2)}$  result in complementary sequences. Moreover, one of them belongs to  $[0, \pi)$  and the other one belongs to  $[\pi, 2\pi)$ . As a result,<sup>8</sup>

$$0 < \theta_1 < \theta_2 < \dots < \theta_N < \pi < \theta_{N+1} < \dots < \theta_{2N} < 2\pi \quad (24)$$

and

$$\hat{\mathbf{x}}_{n+N} = \hat{\mathbf{x}}_n^c, \quad n = 0, 1, \dots, N - 1. \quad (25)$$

This implies that it suffices to identify the  $N$  candidate sequences at  $[0, \theta_N)$  and, then, consider also their complements. Note that we ignore  $[\theta_N, \pi)$  because it corresponds to the complementary sequence  $\hat{\mathbf{x}}_0^c$ .

Similar observations were made in [37]–[41], [69], [70] in the context of PSK, where complementary sequences are equivalent with respect to the metric of interest. This is in contrast to FSK where complementary sequences have in general different metric. Hence, in this work, we have to store both complementary sequences for each interval, while searching for the optimal one.

The proposed algorithm (that includes the above improvements) is presented in Fig. 2. The  $N$  points of decision changes are computed at lines 1–3 and sorted at line 4. At lines 5–12,

<sup>7</sup>Since the constellation is binary, we use the term “complementary sequences” to indicate sequences  $\mathbf{x}$  and  $\mathbf{y}$  that are related by  $\mathbf{y}^c = \mathbf{x}$  (i.e.,  $y_n^c = x_n$ ,  $n = 1, 2, \dots, N$ ) where  $1^c = 2$  and  $2^c = 1$ .

<sup>8</sup>Once more, we assume that  $\theta_i < \theta_{i+1}$ , for any  $i = 1, 2, \dots, N - 1$ , and  $\theta_1 > 0$ . The case where the above assumption does not hold is examined in Footnote 9.

**Algorithm 2** Optimal Noncoherent Binary Orthogonal Detection in Time  $\mathcal{O}(N \log N)$ 


---

**Input:**  $y_1, y_2, \dots, y_N$

- 1: **for**  $n = 1 : N$  **do**
- 2:  $\phi_n := \frac{\pi}{2} + \frac{y_n[1] - y_n[2]}{2} \pmod{\pi}$
- 3: **end for**
- 4:  $(\theta_1, \theta_2, \dots, \theta_N) := \text{sort}(\phi_1, \phi_2, \dots, \phi_N)$
- 5: **if**  $\theta_1 > 0$  **then**
- 6:  $\theta^* := 0$
- 7: **else**
- 8:  $\theta^* := \frac{\theta_N - \pi}{2}$
- 9: **end if**
- 10: **for**  $n = 1 : N$  **do**
- 11:  $\hat{x}_n := \arg \max(\Re\{e^{-j\theta^*} y_n[1]\}, \Re\{e^{-j\theta^*} y_n[2]\})$
- 12: **end for**
- 13:  $\hat{\mathbf{x}}^{\text{comp}} := (\hat{\mathbf{x}})^c$
- 14:  $\text{value}_{\hat{\mathbf{x}}} := y_1[\hat{x}_1] + y_2[\hat{x}_2] + \dots + y_N[\hat{x}_N]$
- 15:  $\text{value}_{\hat{\mathbf{x}}^{\text{comp}}} := y_1[\hat{x}_1^{\text{comp}}] + y_2[\hat{x}_2^{\text{comp}}] + \dots + y_N[\hat{x}_N^{\text{comp}}]$
- 16:  $[\text{ML\_value}, \hat{\mathbf{x}}_{\text{best}}] := \max(|\text{value}_{\hat{\mathbf{x}}}|, |\text{value}_{\hat{\mathbf{x}}^{\text{comp}}}|)$
- 17:  $\hat{\mathbf{x}}^{\text{ML}} := \hat{\mathbf{x}}_{\text{best}}$
- 18: **for**  $i = 1 : N - 1$  **do**
- 19: let  $n$  be the index for which  $\theta_i = \phi_n$  at line 4
- 20:  $\text{value}_{\hat{\mathbf{x}}} := \text{value}_{\hat{\mathbf{x}}} - y_n[\hat{x}_n] + y_n[\hat{x}_n^{\text{comp}}]$
- 21:  $\text{value}_{\hat{\mathbf{x}}^{\text{comp}}} := \text{value}_{\hat{\mathbf{x}}^{\text{comp}}} - y_n[\hat{x}_n^{\text{comp}}] + y_n[\hat{x}_n]$
- 22:  $\hat{x}_n^{\text{comp}} := \hat{x}_n$
- 23:  $\hat{x}_n := \hat{x}_n^{\text{comp}}$
- 24:  $[\text{best\_value}, \hat{\mathbf{x}}_{\text{best}}] := \max(|\text{value}_{\hat{\mathbf{x}}}|, |\text{value}_{\hat{\mathbf{x}}^{\text{comp}}}|)$
- 25: **if**  $\text{best\_value} > \text{ML\_value}$  **then**
- 26:  $\text{ML\_value} := \text{best\_value}$
- 27:  $\hat{\mathbf{x}}^{\text{ML}} := \hat{\mathbf{x}}_{\text{best}}$
- 28: **end if**
- 29: **end for**

**Output:**  $\hat{\mathbf{x}}^{\text{ML}}$

---

Fig. 2. Simplified optimal noncoherent sequence detection algorithm for binary orthogonal modulation and zero-offset FMO coding with complexity  $\mathcal{O}(N \log N)$ .

we identify the initial sequence  $\hat{\mathbf{x}}$  that is optimal in  $C_0$ .<sup>9</sup> At lines 14 and 15, we evaluate the sum in (16) for sequences  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{x}}^c$ , respectively. The two sums are compared against each other with respect to their magnitudes; the best value as well as the corresponding sequence are stored in  $\text{ML\_value}$  and  $\hat{\mathbf{x}}^{\text{ML}}$  at lines 16 and 17, respectively. At lines 18–29, we examine  $\theta_1, \theta_2, \dots, \theta_{N-1}$  serially to find the optimal sequence  $\hat{\mathbf{x}}^{\text{ML}}$ . Specifically, at line 19, we move to the next  $\theta_i$  in the sorted list, which was obtained from some received vector, say  $\mathbf{y}_n$ . At lines 20–23, we update the sums in (16) for sequences  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{x}}^c$  and the value of  $\hat{x}_n$ . At line 24, the two sums are compared against each other and the best one together with the corresponding sequence are stored in  $\text{best\_value}$  and  $\hat{\mathbf{x}}_{\text{best}}$ , respectively. If  $\text{best\_value}$  is greater than the up-to-date  $\text{ML\_value}$ , then we update  $\text{ML\_value}$  and the optimal sequence  $\hat{\mathbf{x}}^{\text{ML}}$  at lines 26 and 27, respectively.

By inspection, the exact computational cost of the algorithm

<sup>9</sup>As in the algorithm of Fig. 1 (see Footnote 6), if  $\phi_n^{(j)} = 0$  for some  $j \in \{1, 2\}$  and  $n \in \{1, 2, \dots, N\}$ , i.e.,  $\theta_1 = 0$ , then we compute  $\hat{\mathbf{x}}_0$  at the mid-point  $\theta^*$  of the interval that precedes  $\theta_1 = 0$ , i.e.,  $\theta^* = \frac{\theta_{2N} - 2\pi}{2}$ . Since  $\theta_{2N} = \theta_N + \pi$  due to (23) and (24), it turns out that  $\theta^* = \frac{\theta_N - \pi}{2}$ , as shown at line 8 of the algorithm in Fig. 2. Otherwise (that is, if  $\phi_n^{(j)} \neq 0$  for any  $j \in \{1, 2\}$  and  $n \in \{1, 2, \dots, N\}$ ),  $\theta_1$  is nonzero and  $\theta^*$  is set to zero, as shown at line 6 of the algorithm in Fig. 2.

Finally, if  $\phi_n^{(j)} = \phi_l^{(k)}$  for some  $j, k \in \{1, 2\}$  and  $n, l \in \{1, 2, \dots, N\}$  with  $n \neq l$ , then  $\theta_i = \theta_{i+1}$  for some  $i \in \{1, 2, \dots, N - 1\}$ . In this case, we act exactly as we did in the algorithm of Fig. 1 (see Footnote 6), i.e., we let the algorithm produce the invalid intermediate sequences  $\hat{\mathbf{x}}_i$  and  $\hat{\mathbf{x}}_i^c$  without increasing its complexity or affecting its optimality.

of Fig. 2, if we count arithmetic operations,<sup>10</sup> equals

$$J_2(N) = N \log_2 N + 17N - 4. \quad (26)$$

Its complexity is dominated by the computational cost of the sorting operation at line 4 which is on the order of  $\mathcal{O}(N \log N)$ .

2) *Optimal algorithm for  $M \geq 2$* : If we fix  $\phi \in [0, 2\pi)$ , then the innermost maximization in (18) splits into independent maximizations for any  $n = 1, 2, \dots, N$ , as

$$\hat{x}_n = \arg \max_{x \in \mathcal{M}} \Re \{ e^{-j\phi} y_n[x] \}. \quad (27)$$

We observe that, for fixed  $\phi$ , (27) is solved by selecting the largest value of  $\Re \{ e^{-j\phi} y_n \}$ . As  $\phi$  scans  $[0, 2\pi)$ , the decision  $\hat{x}_n$  may change only when, for some  $k, l \in \mathcal{M}$  with  $k \neq l$ ,

$$\begin{aligned} \Re \{ e^{-j\phi} y_n[k] \} &= \Re \{ e^{-j\phi} y_n[l] \} \\ \Leftrightarrow \Re \{ e^{-j\phi} (y_n[k] - y_n[l]) \} &= 0 \\ \Leftrightarrow \cos(\phi - \underbrace{\angle y_n[k] - y_n[l]}_{\phi_{n,\{k,l\}}^{(1)}, \phi_{n,\{k,l\}}^{(2)}}) &= 0 \\ \Leftrightarrow \phi &= \pm \frac{\pi}{2} + \underbrace{\angle y_n[k] - y_n[l]}_{\phi_{n,\{k,l\}}^{(1)}, \phi_{n,\{k,l\}}^{(2)}} \pmod{2\pi}. \end{aligned} \quad (28)$$

Hence, as  $\phi$  scans  $[0, 2\pi)$ , the decision on the sequence  $\hat{\mathbf{x}}$  may change only at some of the points defined in (28), for any  $k, l \in \mathcal{M}$  with  $k \neq l$  and  $n = 1, 2, \dots, N$ . Although (28) produces  $2N \binom{M}{2} = M(M-1)N$  such points, it turns out that the decision  $\hat{\mathbf{x}}$  changes at only (at most)  $2(M-1)N$  points. This is stated in the following proposition.

*Proposition 1*: For  $M \geq 2$ , there exist at most  $2(M-1)N$  changes of the sequence decision  $\hat{\mathbf{x}}$  in the interval  $[0, 2\pi)$ .

*Proof*:

Since, for a given  $\phi \in [0, 2\pi)$ , the  $N$  maximizations in (18) are independent of each other, it suffices to restrict our attention to a single symbol  $x_n$  and show that the decision  $\hat{x}_n$  changes at most  $2(M-1)$  times in the interval  $[0, 2\pi)$ .

Consider first the interval  $[0, \pi)$ . As  $\phi$  scans  $[0, \pi)$ , the decision  $\hat{x}_n$  changes at  $K$  points given by (28), say  $\phi_{n_1}, \phi_{n_2}, \dots, \phi_{n_K}$ , where, without loss of generality,<sup>11</sup>

$$0 < \phi_{n_1} < \phi_{n_2} < \dots < \phi_{n_K} < \pi. \quad (29)$$

That is, the interval  $[0, \pi)$  is partitioned into  $K+1$  successive intervals

$$C_{n_0} = (0, \phi_{n_1}), C_{n_1} = (\phi_{n_1}, \phi_{n_2}), \dots, C_{n_K} = (\phi_{n_K}, \pi) \quad (30)$$

in such a way that the decision in favor of  $x_n$  (i) is constant in each interval and (ii) is different over successive intervals. Let  $\hat{x}_{n_0}, \hat{x}_{n_1}, \dots, \hat{x}_{n_K}$  be these  $K+1$  decisions on  $x_n$ .

Since the decision on  $x_n$  is made using (27), we define the metric function in (27) with respect to symbol  $x_n$  as

$$\mu_n(\phi; x) = \Re \{ e^{-j\phi} y_n[x] \}, \quad \phi \in [0, 2\pi), \quad x \in \mathcal{M}. \quad (31)$$

<sup>10</sup>Complex addition, subtraction, multiplication, and division, magnitude, angle, modulo, and maximization operations, and logical comparison account for a single arithmetic operation.

<sup>11</sup>We again assume that  $\phi_{n_1} > 0$ . If  $\phi_{n_1} = 0$ , then we let  $\phi$  scan the interval  $[\theta^*, \pi + \theta^*)$  where  $\theta^* < 0$  is an appropriately selected point in the interval that precedes  $\phi_{n_1} = 0$ , as described in Footnote 6.

Consider now an arbitrary value of  $k \in \{0, 1, 2, \dots, K-1\}$ . Since  $\hat{x}_n = \hat{x}_{n_k}$  for any  $\phi \in C_{n_k}$  and  $\hat{x}_n = \hat{x}_{n_{k+1}}$  for any  $\phi \in C_{n_{k+1}}$ , it is implied that

$$\begin{aligned} \mu_n(\phi; \hat{x}_{n_k}) &> \mu_n(\phi; \hat{x}_{n_{k+1}}), \text{ if } \phi \in C_{n_k}, \\ \mu_n(\phi; \hat{x}_{n_k}) &= \mu_n(\phi; \hat{x}_{n_{k+1}}), \text{ if } \phi = \phi_{n_{k+1}}, \\ \mu_n(\phi; \hat{x}_{n_k}) &< \mu_n(\phi; \hat{x}_{n_{k+1}}), \text{ if } \phi \in C_{n_{k+1}}. \end{aligned} \quad (32)$$

The latter inequality will hold true as long as  $\phi$  does not meet any value  $\phi' \neq \phi_{n_{k+1}}$  such that  $\mu_n(\phi'; \hat{x}_{n_k}) = \mu_n(\phi'; \hat{x}_{n_{k+1}})$ . By (28), there is only one such a point, namely  $\phi_{n_{k+1}} + \pi$ , which, however, lies outside  $[0, \pi)$ . Hence,

$$\mu_n(\phi; \hat{x}_{n_k}) < \mu_n(\phi; \hat{x}_{n_{k+1}}), \quad \forall \phi \in (\phi_{n_{k+1}}, \pi). \quad (33)$$

Similarly, considering an arbitrary value of  $k \in \{1, 2, \dots, K\}$  and using the same arguments, we can show that

$$\mu_n(\phi; \hat{x}_{n_k}) < \mu_n(\phi; \hat{x}_{n_{k-1}}), \quad \forall \phi \in (0, \phi_{n_k}). \quad (34)$$

From (33), it is implied that  $\hat{x}_{n_0}$ , i.e., the decision in  $C_{n_0}$ , cannot reappear in any other interval among  $C_{n_1}, C_{n_2}, \dots, C_{n_K}$  in  $[0, \pi)$ . Similarly, from (34), it is implied that  $\hat{x}_{n_K}$  appears only in  $C_{n_K}$ . Finally, for any  $k = 1, 2, \dots, K-1$ , (33) and (34) imply that the decision  $\hat{x}_{n_k}$  cannot appear in  $(0, \phi_{n_k})$  or  $(\phi_{n_{k+1}}, \pi)$ . Hence, it appears only in  $(\phi_{n_k}, \phi_{n_{k+1}})$ , which is  $C_{n_k}$  in (30). Therefore, the  $K+1$  intervals in (30) correspond to distinct decisions, i.e.,  $\hat{x}_{n_k} \neq \hat{x}_{n_j}$  if  $k, j \in \{0, 1, \dots, K\}$  with  $k \neq j$ . Since there are at most  $M$  available values for  $x_n$ , we conclude that  $K+1 \leq M$ , i.e.,

$$K \leq M - 1. \quad (35)$$

Consider now the interval  $[\pi, 2\pi)$ . If the decision  $\hat{x}_n$  changes at  $K'$  points as  $\phi$  scans  $[\pi, 2\pi)$ , then, we can similarly show that

$$K' \leq M - 1. \quad (36)$$

As  $\phi$  scans the entire circle  $[0, 2\pi)$ , the decision  $\hat{x}_n$  changes exactly  $K+K'$  times. From (35) and (36), we obtain  $K+K' \leq 2M-2 = 2(M-1)$ . That is, the decision  $\hat{x}_n$  changes at most  $2(M-1)$  times. Therefore, the sequence decision  $\hat{\mathbf{x}}$  changes at most  $2(M-1)N$  times.  $\square$

The above proposition states that it suffices to check at most  $2(M-1)N$  points where the sequence decision changes. When the points have been determined, the remaining process resembles to the algorithm of case  $M = 2$ . Specifically, after the identification of the points that correspond to actual decision changes, we seek the sequence  $\hat{\mathbf{x}}$  obtained at each point  $\theta_i$  that gives the largest metric in (16). In the case of  $M > 2$ , sequence  $\hat{\mathbf{x}}(\phi)$  is not necessarily complementary with  $\hat{\mathbf{x}}(\phi + \pi)$  and, thus, we need to seek the optimal sequence within the entire interval  $[0, 2\pi)$ .

The proposed algorithm for optimal noncoherent  $M$ -ary orthogonal sequence detection is depicted in Fig. 3. By inspection, the exact computational cost of the algorithm, if we count arithmetic operations, is upper bounded by

$$\begin{aligned} J_M(N) &= 2(M-1)N \log_2(2(M-1)N) \\ &+ M(M-1)N \log_2(M(M-1)) \\ &+ \left( \frac{1}{4}M^4 + \frac{1}{2}M^3 + \frac{35}{4}M^2 + \frac{1}{2}M - 5 \right) N. \end{aligned} \quad (37)$$

**Algorithm 3** Optimal Noncoherent  $M$ -ary Orthogonal Detection in Time  $\mathcal{O}(N \log N)$ 

```

Input:  $y_1, y_2, \dots, y_N$ 
1: for  $n = 1 : N$  do
2:    $\phi_n := []$ 
3:    $\mathbf{S}_n := []$ 
4:   for  $k, l \in \mathcal{M}$  such that  $k \neq l$  do
5:      $\phi_n^{(1)} := +\frac{\pi}{2} + \sqrt{y_n[k] - y_n[l]} \pmod{2\pi}$ 
6:      $\phi_n^{(2)} := -\frac{\pi}{2} + \sqrt{y_n[k] - y_n[l]} \pmod{2\pi}$ 
7:      $\phi_n := [\phi_n, \phi_n^{(1)}, \phi_n^{(2)}]$ 
8:      $\mathbf{S}_n := [\mathbf{S}_n, \{k, l\}, \{k, l\}]$ 
9:   end for
10: end for
11:  $\phi := []$ 
12:  $\mathbf{I} := []$ 
13:  $\mathbf{S} := []$ 
14: for  $n = 1 : N$  do
15:    $[\phi_n, \text{sort\_indices}] := \text{sort}(\phi_n)$ 
16:    $\mathbf{S}_n := \mathbf{S}_n[\text{sort\_indices}]$ 
17:   if  $\phi_n[1] > 0$  then
18:      $\theta^* := 0$ 
19:   else
20:      $\theta^* := \frac{\phi_n[M(M-1)] - 2\pi}{2}$ 
21:   end if
22:    $\hat{x}_n := \arg \max \{ \Re \{ e^{-j\theta^*} y_n[1] \}, \Re \{ e^{-j\theta^*} y_n[2] \}, \dots, \Re \{ e^{-j\theta^*} y_n[M] \} \}$ 
23:    $m := \hat{x}_n$ 
24:    $j := 1$ 
25:   while  $j \leq M(M-1)$  do
26:      $j' := \max\{j, j+1, \dots, M(M-1)\}$  such that  $\phi_n[j'] = \phi_n[j]$ 
27:     if  $j' = j$  then
28:       if  $m \in \mathbf{S}_n[j]$  then
29:          $m :=$  the unique element of set  $\mathbf{S}_n[j] \setminus \{m\}$ 
30:          $\phi := [\phi, \phi_n[j]]$ 
31:          $\mathbf{I} := [\mathbf{I}, n]$ 
32:          $\mathbf{S} := [\mathbf{S}, \mathbf{S}_n[j]]$ 
33:       end if
34:     else
35:        $\mathcal{P} := \bigcup_{i=j}^{j'} \mathbf{S}_n[i]$ 
36:       if  $\mathcal{P} \cap \{m\} \neq \emptyset$  then
37:         if  $j' < M(M-1)$  then
38:            $\phi^* := \frac{\phi_n[j'] + \phi_n[j'+1]}{2}$ 
39:         else
40:            $\phi^* := \frac{\phi_n[M(M-1)] - 2\pi}{2}$ 
41:         end if
42:          $m' := \arg \max \{ \Re \{ e^{-j\phi^*} y_n[1] \}, \Re \{ e^{-j\phi^*} y_n[2] \}, \dots, \Re \{ e^{-j\phi^*} y_n[M] \} \}$ 
43:          $\phi := [\phi, \phi_n[j']]$ 
44:          $\mathbf{I} := [\mathbf{I}, n]$ 
45:          $\mathbf{S} := [\mathbf{S}, \{m, m'\}]$ 
46:          $m := m'$ 
47:       end if
48:     end if
49:      $j := j' + 1$ 
50:   end while
51: end for
52:  $[\theta, \text{sort\_indices}] := \text{sort}(\phi)$ 
53:  $\mathbf{I} := \mathbf{I}[\text{sort\_indices}]$ 
54:  $\mathbf{S} := \mathbf{S}[\text{sort\_indices}]$ 
55:  $\hat{x}^{\text{ML}} := \hat{x}$ 
56:  $\text{value\_}\hat{x} := y_1[\hat{x}_1] + y_2[\hat{x}_2] + \dots + y_N[\hat{x}_N]$ 
57:  $\text{ML\_value} := |\text{value\_}\hat{x}|$ 
58: for  $i = 1 : \text{length}(\theta)$  do
59:    $n := \mathbf{I}[i]$ 
60:    $m := \hat{x}_n$ 
61:    $m' :=$  the unique element of set  $\mathcal{S}[i] \setminus \{m\}$ 
62:    $\text{value\_}\hat{x} := \text{value\_}\hat{x} - y_n[m] + y_n[m']$ 
63:    $\hat{x}_n := m'$ 
64:    $\text{best\_value} := |\text{value\_}\hat{x}|$ 
65:   if  $\text{best\_value} > \text{ML\_value}$  then
66:      $\text{ML\_value} := \text{best\_value}$ 
67:      $\hat{x}^{\text{ML}} := \hat{x}$ 
68:   end if
69: end for
Output:  $\hat{x}^{\text{ML}}$ 
    
```

 Fig. 3. Optimal noncoherent sequence detection algorithm for  $M$ -ary orthogonal modulation with complexity  $\mathcal{O}(N \log N)$ .

The overall complexity of the algorithm is dominated by the sorting operation at line 52 and, thus, the worst-case complexity of the algorithm is  $\mathcal{O}(2(M-1)N \log_2 2(M-1)N) =$

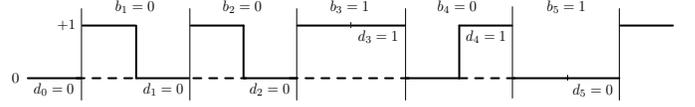


Fig. 4. Transmitted waveform in zero-offset FM0 coding.

$\mathcal{O}(N \log N)$ .

### III. ZERO-OFFSET FM0 CODING

FM0 (also called biphas-space or differential biphas coding [46]) is a line-coding technique that is used in the current RFID communications standard. In zero-offset FM0, the signal level can take two possible values; namely, 0 and 1. Specifically, the level changes at the middle of the bit period for bit 0, whereas for bit 1 it remains constant. Moreover, it always changes at the beginning of every bit period, as can be seen in Fig. 4, and, thus, the signals from one bit interval to another are not independent (i.e., FM0 induces memory). As a result, four possible transmitted waveforms can be generated which are depicted in Fig. 4 and can be represented in vector form as  $[0 \ 0]^T$ ,  $[0 \ 1]^T$ ,  $[1 \ 0]^T$ , and  $[1 \ 1]^T$ .

Consider, for example, the transmission of the  $n$ th information bit during the  $n$ th period. We denote by  $d_n \in \{0, 1\}$  the signal level at the end of  $n$ th bit period. Then, the signal level  $d_{n-1}$  at the end of the preceding period will change to  $d_{n-1}^c$  during the first half of the  $n$ th period.<sup>12</sup> For the second half, it will change to  $d_n = d_{n-1}$  if the information bit is 0, or it will remain  $d_n = d_{n-1}^c$  if the information bit is 1. This can be compactly expressed as

$$d_n = d_{n-1} \oplus b_n, \quad n = 1, 2, \dots, N, \quad (38)$$

where  $\oplus$  denotes exclusive-OR operation. Hence, during the  $n$ th bit period, the signal level takes the values  $(d_{n-1}^c, \underbrace{d_{n-1} \oplus b_n}_{d_n})$ .

Assuming a sequence of  $N$  information bits  $b_1, b_2, \dots, b_N \in \{0, 1\}$ , the corresponding transmitted sequence is

$$\begin{array}{c}
 \longrightarrow \left| \begin{array}{c} \text{bit } b_1 \\ \leftarrow d_0^c \quad \left| \begin{array}{c} d_0 \oplus b_1 \\ \hline d_1 \end{array} \right| \leftarrow d_1^c \quad \left| \begin{array}{c} d_1 \oplus b_2 \\ \hline d_2 \end{array} \right| \leftarrow d_2^c \quad \left| \begin{array}{c} d_2 \oplus b_3 \\ \hline d_3 \end{array} \right| \dots \end{array} \right. \\
 \dots \left| \begin{array}{c} \text{bit } b_N \\ \leftarrow d_{N-1}^c \quad \left| \begin{array}{c} d_{N-1} \oplus b_N \\ \hline d_N \end{array} \right| \leftarrow d_N^c \end{array} \right. \quad (39)
 \end{array}$$

or, in vector form,

$$[d_0, d_0^c, d_1, d_1^c, \dots, d_N, d_N^c]^T = \underbrace{\begin{bmatrix} \mathbf{e}_{d_0} \\ \mathbf{e}_{d_1} \\ \vdots \\ \mathbf{e}_{d_N} \end{bmatrix}}_{\mathbf{d}} \quad (40)$$

<sup>12</sup>In this section, signal values 0 and 1 are considered complementary, i.e.,  $0^c = 1$  and  $1^c = 0$ .

where

$$d_0 \in \{0, 1\}, \quad (41)$$

$$d_n = d_{n-1} \oplus b_n = d_0 \oplus b_1 \oplus b_2 \oplus \dots \oplus b_n, \quad n = 1, 2, \dots, N, \quad (42)$$

$$\mathbf{e}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (43)$$

Hence, zero-offset FM0 coding is equivalent to differential BFSK modulation. For instance, if  $b_1 = 0$ ,  $b_2 = 0$ ,  $b_3 = 1$ ,  $b_4 = 0$ , and  $b_5 = 1$  and we use zero-offset FM0 coding with  $d_0 = 0$ , then we obtain  $d_1 = 0$ ,  $d_2 = 0$ ,  $d_3 = 1$ ,  $d_4 = 1$ , and  $d_5 = 0$ , resulting in the waveform of Fig. 4.

The transmitted vector is  $\sqrt{P}\mathbf{d}$  where  $P$  is the signal strength. Upon transmission over a flat-fading channel whose coherence time spans at least  $N + 1$  symbols [46], [50], [51], [55], the received vector is

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \sqrt{P}h\mathbf{d} + \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_N \end{bmatrix} \quad (44)$$

where  $\mathbf{y}_n = \sqrt{P}h\mathbf{e}_{d_n} + \mathbf{w}_n$ ,  $n = 0, 1, \dots, N$ ,  $h$  is the complex channel coefficient, and  $\mathbf{w}_0, \mathbf{w}_1, \dots, \mathbf{w}_N$  is a sequence of independent and identically distributed  $2 \times 1$  vectors that represent zero-mean additive white circularly symmetric complex Gaussian noise, i.e.,  $\mathbf{w}_n \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_2)$ ,  $n = 0, 1, \dots, N$ .

The optimal noncoherent zero-offset FM0 sequence detector maximizes the conditional pdf of  $\mathbf{y}$  given  $\mathbf{d}$ , that is,

$$\hat{\mathbf{d}}^{\text{ML}} = \arg \max_{\mathbf{d} \in \mathcal{I}_2^{N+1}} f(\mathbf{y}|\mathbf{d}). \quad (45)$$

The optimization problem in (45) is equivalent to optimal noncoherent BFSK detection of (9). Hence, for Rayleigh fading (i.e.,  $h \sim \mathcal{CN}(0, \sigma_h^2)$ ) [50], [51], [55], from (15) we obtain

$$\hat{\mathbf{d}}^{\text{ML}} = \arg \max_{\mathbf{d} \in \mathcal{I}_2^{N+1}} |\mathbf{d}^T \mathbf{y}|. \quad (46)$$

As with BFSK, (46) also offers GLRT-optimal sequence detection when the channel is non-Rayleigh or unknown. To find the optimal solution in (46), the algorithm for BFSK, presented in Fig. 2, can be directly employed to the sequence of received vectors  $\mathbf{y}_0, \mathbf{y}_1, \dots, \mathbf{y}_N$  to obtain the sequence  $\hat{\mathbf{d}}^{\text{ML}}$  with complexity  $\mathcal{O}((N+1)\log(N+1)) = \mathcal{O}(N\log N)$ . Finally, after identification of the optimal sequence  $\hat{\mathbf{d}}^{\text{ML}}$ , the optimal information data sequence  $\hat{\mathbf{b}}^{\text{ML}}$  is obtained by plain differential decoding, i.e.,

$$\hat{b}_n^{\text{ML}} = \hat{d}_n^{\text{ML}} \oplus \hat{d}_{n-1}^{\text{ML}}, \quad n = 1, 2, \dots, N. \quad (47)$$

#### IV. ANTIPODAL FM0 CODING

##### A. Signal Model and Optimal Sequence Detection

In antipodal FM0 coding, the transmission process adopts the same principles as in zero-offset FM0 coding. The only difference is that the signal level takes the values 1 and  $-1$  (instead of 0 and 1). The four possible transmitted waveforms are depicted in Fig. 5 and can be represented in vector form as  $[1 \ 1]^T$ ,  $[1 \ -1]^T$ ,  $[-1 \ 1]^T$ , and  $[-1 \ -1]^T$ .

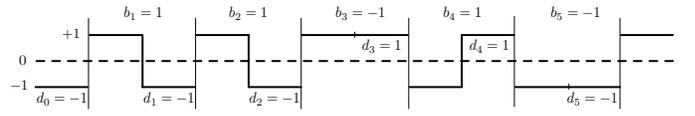


Fig. 5. Transmitted waveform in antipodal FM0 coding.

Similarly to zero-offset FM0 coding, it is straightforward to show that  $d_n = d_{n-1}b_n$ . Hence, during the  $n$ th bit period, the signal level takes the values  $(-d_{n-1}, \underbrace{d_{n-1}b_n}_{d_n})$ . For a sequence of  $N$  information bits  $b_1, b_2, \dots, b_N \in \{\pm 1\}$ , the corresponding transmitted sequence is

$$\begin{array}{c} \xrightarrow{\quad} \left| \begin{array}{c} \xleftarrow{\text{bit } b_1} \\ -d_0 \quad \left| \begin{array}{c} d_0 b_1 \\ d_1 \end{array} \right| \\ \xleftarrow{\text{bit } b_2} \\ -d_1 \quad \left| \begin{array}{c} d_1 b_2 \\ d_2 \end{array} \right| \\ \xleftarrow{\text{bit } b_3} \\ -d_2 \quad \left| \begin{array}{c} d_2 b_3 \\ d_3 \end{array} \right| \\ \dots \end{array} \right. \dots \\ \dots \left| \begin{array}{c} \xleftarrow{\text{bit } b_N} \\ -d_{N-1} \quad \left| \begin{array}{c} d_{N-1} b_N \\ d_N \end{array} \right| \\ -d_N \end{array} \right. \end{array} \quad (48)$$

or, in vector form,

$$[d_0, -d_0, d_1, -d_1, \dots, d_N, -d_N]^T = \underbrace{\begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_N \end{bmatrix}}_{\mathbf{d}} \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (49)$$

where

$$d_0 \in \{\pm 1\}, \quad (50)$$

$$d_n = d_{n-1}b_n = d_0 b_1 b_2 \dots b_n, \quad n = 1, 2, \dots, N, \quad (51)$$

and  $\otimes$  denotes Kronecker-product operation. Hence, antipodal FM0 coding is equivalent to differential antipodal modulation, as shown in [46]. For instance, if  $b_1 = +1$ ,  $b_2 = +1$ ,  $b_3 = -1$ ,  $b_4 = +1$ , and  $b_5 = -1$  and we use antipodal FM0 line coding with  $d_0 = -1$ , then we obtain  $d_1 = -1$ ,  $d_2 = -1$ ,  $d_3 = +1$ ,  $d_4 = +1$ , and  $d_5 = -1$ , resulting in the waveform of Fig. 5.

The transmitted signal vector is  $\sqrt{\frac{P}{2}}\mathbf{d} \otimes \begin{bmatrix} 1 \\ -1 \end{bmatrix}$  where  $P$  is the signal strength. Upon transmission over a flat-fading channel whose coherence time spans at least  $N + 1$  symbols, the optimal (coherent or noncoherent) detector correlates the downconverted received sequence with the pulse  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$  at the bit rate (with an offset by half bit period to match the corresponding transmitted sequence), resulting in the  $(N + 1) \times 1$  vector

$$\mathbf{y} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{bmatrix} = \sqrt{P}h\mathbf{d} + \mathbf{w} \quad (52)$$

where  $h$  is the complex channel coefficient and  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{N+1})$  denotes zero-mean additive white circularly symmetric complex Gaussian noise.

The optimal noncoherent antipodal FM0 sequence detector maximizes the conditional pdf of  $\mathbf{y}$  given  $\mathbf{d}$ , that is,

$$\hat{\mathbf{d}}^{\text{ML}} = \arg \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} f(\mathbf{y}|\mathbf{d}). \quad (53)$$

It can be shown that, for Rayleigh fading, the ML sequence detection optimization problem can be expressed as

$$\hat{\mathbf{d}}^{\text{ML}} = \arg \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} |\mathbf{d}^T \mathbf{y}| \quad (54)$$

by following the same derivation steps as in Section II-A (eq's (8)–(15)). For non-Rayleigh or unknown channel, the GLRT rule for antipodal FM0 coding admits the same optimization problem, i.e.,

$$\hat{\mathbf{d}}^{\text{GLRT}} = \arg \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} |\mathbf{d}^T \mathbf{y}| \quad (55)$$

by following the same derivation as in eq. (17). After identification of the optimal sequence  $\hat{\mathbf{d}}^{\text{ML}}$ , the optimal information data sequence  $\hat{\mathbf{b}}^{\text{ML}}$  is obtained by plain differential decoding, i.e.,

$$\hat{b}_n^{\text{ML}} = \hat{d}_n^{\text{ML}} \hat{d}_{n-1}^{\text{ML}}, \quad n = 1, 2, \dots, N. \quad (56)$$

### B. Log-linear-complexity Optimal Detection

An efficient algorithm that computes the solution of (55) with log-linear complexity was developed in [37], [38].<sup>13</sup> For completeness, we present it shortly in this subsection.

As in the development of the optimal algorithm for BFSK detection (Section II-B), we use the fact that

$$\begin{aligned} \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} |\mathbf{d}^T \mathbf{y}| &= \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} |d_0 y_0 + d_1 y_1 + \dots + d_N y_N| \\ &= \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} \max_{\phi \in [0, 2\pi)} \Re \{ e^{-j\phi} (d_0 y_0 + d_1 y_1 + \dots + d_N y_N) \} \\ &= \max_{\phi \in [0, 2\pi)} \max_{\mathbf{d} \in \{\pm 1\}^{N+1}} \left\{ d_0 \Re \{ e^{-j\phi} y_0 \} + d_1 \Re \{ e^{-j\phi} y_1 \} + \dots \right. \\ &\quad \left. \dots + d_N \Re \{ e^{-j\phi} y_N \} \right\}. \end{aligned} \quad (57)$$

For a given point  $\phi \in [0, 2\pi)$ , the inner maximization in (57) is separable for each  $d_n$  and, thus, splits into independent maximizations for any  $n = 0, 1, \dots, N$ , as

$$\begin{aligned} \hat{d}_n &= \arg \max_{d_n \in \{\pm 1\}} d_n \Re \{ e^{-j\phi} y_n \} \\ &\Leftrightarrow \Re \{ e^{-j\phi} y_n \} \stackrel{\hat{d}_n = +1}{\geq} -\Re \{ e^{-j\phi} y_n \} \\ &\Leftrightarrow \Re \{ e^{-j\phi} y_n \} \stackrel{\hat{d}_n = +1}{\geq} 0 \Leftrightarrow \cos(\phi - \underline{y}_n) \stackrel{\hat{d}_n = +1}{\geq} 0. \end{aligned} \quad (58)$$

According to (58), as  $\phi$  scans  $[0, 2\pi)$ , the decision  $\hat{d}_n$  changes only when

$$\phi = \pm \frac{\pi}{2} + \underline{y}_n \pmod{2\pi}. \quad (59)$$

For any  $\phi \in [0, \pi)$ , we observe that the candidate sequences that we obtain at  $\phi$  and  $\phi + \pi$  are opposite, since  $\cos(\phi - \underline{y}_n) = -\cos(\phi + \pi - \underline{y}_n)$ ,  $n = 0, 1, \dots, N$ ,

in (58). Since opposite sequences result in the same metric value in (54), it suffices to restrict our search in  $\phi \in [0, \pi)$ . Hence, we keep the  $N + 1$  points in (59) that belong to  $[0, \pi)$  and define them as

$$\phi_n = \frac{\pi}{2} + \underline{y}_n \pmod{\pi}, \quad n = 0, 1, \dots, N. \quad (60)$$

Subsequently, we sort the  $N + 1$  points through<sup>14</sup>

$$(\theta_0, \theta_1, \dots, \theta_N) = \text{sort}(\phi_0, \phi_1, \dots, \phi_N). \quad (61)$$

Then, the decision  $\hat{\mathbf{d}}$  remains constant in each one of the  $N + 1$  intervals

$$C_0 = (0, \theta_0), C_1 = (\theta_0, \theta_1), \dots, C_N = (\theta_{N-1}, \theta_N). \quad (62)$$

We note that we ignore the interval  $(\theta_N, \pi)$  because it corresponds to the opposite sequence of the one in  $C_0$ .

Our objective now becomes the identification of the  $N + 1$  candidate sequences

$$\hat{\mathbf{d}}_0, \hat{\mathbf{d}}_1, \dots, \hat{\mathbf{d}}_N \quad (63)$$

that are associated with the intervals  $C_0, C_1, \dots, C_N$ , respectively, in  $[0, \theta_N)$ . We observe that sequences that correspond to adjacent intervals differ in exactly one element. For example,  $\hat{\mathbf{d}}_0$  and  $\hat{\mathbf{d}}_1$  differ in the element that produced  $\theta_0$ . Hence, we propose to (i) identify  $\hat{\mathbf{d}}_0$  at  $\phi = 0$  through (58) and (ii) successively visit the angles  $\theta_0, \dots, \theta_{N-1}$  to produce the remaining  $N$  sequences, evaluate their metric in (54), and track the best sequence and its metric. Note that, at each point  $\theta_i$ , the new sequence  $\hat{\mathbf{d}}_{i+1}$  is produced with constant complexity by changing only one element of the preceding sequence  $\hat{\mathbf{d}}_i$ . The metric of  $\hat{\mathbf{d}}_{i+1}$  is obtained with constant complexity by simply updating the metric of  $\hat{\mathbf{d}}_i$  with respect to the single element that changed at  $\theta_i$ .

The optimal algorithm for ML sequence detection of antipodal FM0 is illustrated in Fig. 6. The overall complexity is dominated by the computational cost of the sorting operation at line 4 which is on the order of  $\mathcal{O}((N + 1)\log_2(N + 1)) = \mathcal{O}(N \log N)$ .

Comparing the two algorithms in Figs. 2 and 6, we observe that the second one constructively builds and compares  $N$  sequences, instead of  $2N$  sequences that are built by the first algorithm. This happens because the algorithm in Fig. 2 has to track both a candidate sequence  $\hat{\mathbf{x}}$  and its complementary sequence  $\hat{\mathbf{x}}^c$ . These two sequences have different metrics, as explained in Section II-B1. On the contrary, the algorithm in Fig. 6 calculates the metric of only half of the  $2N$  sequences and avoids examining their opposite ones, since opposite sequences result in the same metric value in (54).

<sup>13</sup>The algorithm of [37], [38] reappeared in [41]. The principles of the algorithm in [37], [38] were followed also in [69] to develop a quadratic-complexity optimal algorithm.

<sup>14</sup>Again, we assume that  $\phi_n \neq \phi_l$  and  $\phi_n \neq 0$  for any  $n, l \in \{0, 1, \dots, N\}$  with  $n \neq l$ . If  $\phi_n = 0$  for some  $n \in \{0, 1, \dots, N\}$ , then we modify our search to the interval  $[\theta^*, \pi + \theta^*)$ , where  $\theta^* = \frac{\theta_{N-\pi}}{2} < 0$ , exactly as we did in Footnote 9 for the algorithm of Fig. 2. Finally, if  $\phi_n = \phi_l$  for some  $n, l \in \{0, 1, \dots, N\}$  with  $n \neq l$ , then we let the algorithm produce an invalid intermediate sequence without increasing its complexity or affecting its optimality, exactly as we did in Footnote 9 for the algorithm of Fig. 2.

**Algorithm 4** Optimal Noncoherent Antipodal FM0 Decoding in Time  $\mathcal{O}(N \log N)$ 

**Input:**  $y_0, y_1, \dots, y_N$   
 1: **for**  $n = 0 : N$  **do**  
 2:    $\hat{\phi}_n := \frac{\pi}{2} + \angle y_n \pmod{\pi}$   
 3: **end for**  
 4:  $(\theta_0, \theta_1, \dots, \theta_N) := \text{sort}(\hat{\phi}_0, \hat{\phi}_1, \dots, \hat{\phi}_N)$   
 5: **if**  $\theta_0 > 0$  **then**  
 6:    $\theta^* := 0$   
 7: **else**  
 8:    $\theta^* := \frac{\theta_N - \pi}{2}$   
 9: **end if**  
 10: **for**  $n = 0 : N$  **do**  
 11:    $\hat{d}_n := \text{sign}(\Re\{e^{-j\theta^*} y_n\})$   
 12: **end for**  
 13:  $\hat{\mathbf{d}}^{\text{ML}} := \hat{\mathbf{d}}$   
 14:  $\text{value}_{\hat{\mathbf{d}}} := \hat{d}_0 y_0 + \hat{d}_1 y_1 + \dots + \hat{d}_N y_N$   
 15:  $\text{ML\_value} := |\text{value}_{\hat{\mathbf{d}}}|$   
 16: **for**  $i = 0 : N - 1$  **do**  
 17:   let  $n$  be the index for which  $\theta_i = \phi_n$  at line 4  
 18:    $\text{value}_{\hat{\mathbf{d}}} := \text{value}_{\hat{\mathbf{d}}} - 2\hat{d}_n y_n$   
 19:    $\hat{d}_n := -\hat{d}_n$   
 20:   **if**  $|\text{value}_{\hat{\mathbf{d}}}| > \text{ML\_value}$  **then**  
 21:      $\text{ML\_value} := |\text{value}_{\hat{\mathbf{d}}}|$   
 22:      $\hat{\mathbf{d}}^{\text{ML}} := \hat{\mathbf{d}}$   
 23:   **end if**  
 24: **end for**  
**Output:**  $\hat{\mathbf{d}}^{\text{ML}}$

Fig. 6. Optimal noncoherent sequence detection algorithm for antipodal FM0 coding with complexity  $\mathcal{O}(N \log N)$ .

## V. SIMULATION RESULTS

We consider BFSK transmissions through a Rayleigh flat-fading channel. In Fig. 7, we plot the bit error rate (BER) of the ML noncoherent sequence detector as a function of the received signal-to-noise ratio (SNR) which is given by

$$\text{SNR} = \frac{P\sigma_h^2}{\sigma_w^2}, \quad (64)$$

for sequence length  $N = 1, 2, 10, \text{ and } 100$ .<sup>15</sup> Especially for the case of the noncoherent energy detector (i.e.,  $N = 1$ ), we also plot the theoretical expression for the BER in Rayleigh flat fading, given by [28]

$$\text{BER} = \frac{1}{\text{SNR} + 2}. \quad (65)$$

As a reference, we include the BER of the conventional ML coherent detector with perfect channel knowledge. We observe that, as the sequence length increases, the noncoherent detector approaches the coherent one in terms of BER. Moreover, the BER of the conventional energy detector (i.e.,  $N = 1$ ) is 3–5dB far from the coherent one; as the sequence length  $N$  increases, the BER gap decreases to zero.

To demonstrate the rate of convergence to coherent detection performance, in Fig. 8, we set the SNR to 10dB and plot the BER of the ML noncoherent detector as a function of the sequence length  $N$ . We include the BER of the ML coherent detector, as a reference. We note that the BER of the noncoherent scheme with  $N = 100$  is nearly equal to the BER of the coherent one with perfect channel knowledge. In the same figure, we plot the computational cost of the ML noncoherent detector implemented by both the proposed algorithm and the conventional exhaustive-search approach, as

<sup>15</sup>Since the channel is Rayleigh distributed, the ML and GLRT noncoherent detectors coincide.

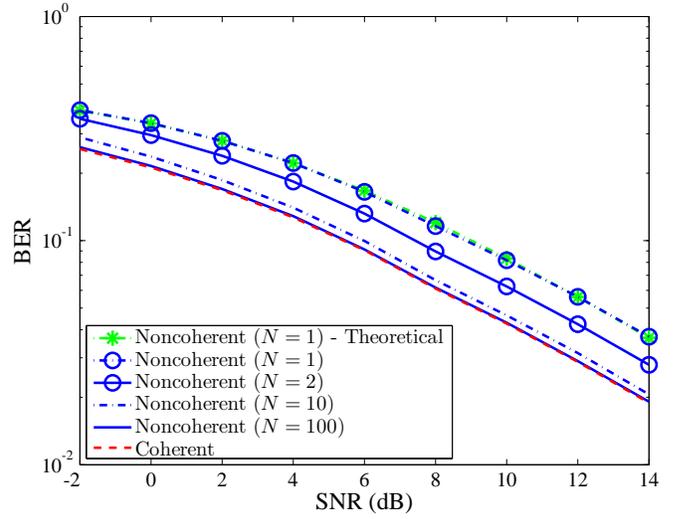


Fig. 7. BER versus bit SNR of ML/GLRT noncoherent BFSK detection with sequence length  $N = 1, 2, 10, 100$  and ML coherent BFSK detection with perfect channel knowledge.

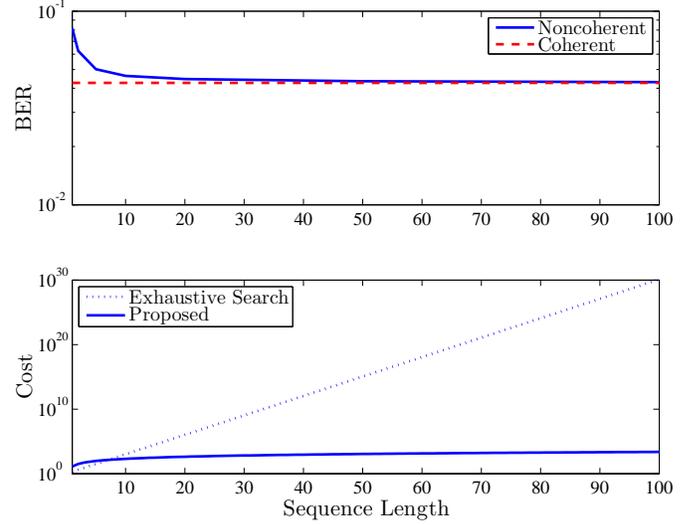


Fig. 8. BER and computational cost of ML/GLRT noncoherent BFSK detection versus sequence length  $N$  for SNR = 10dB.

a function of the sequence length  $N$ . We recall that the cost of the proposed algorithm equals  $J_2(N)$ , given by (26), while the cost of the conventional exhaustive-search approach is  $2^N$ .

Finally, in Figs. 9 and 10, we repeat the above studies for 4FSK modulation and make similar observations for its symbol error rate (SER). In Fig. 9, for the case of the noncoherent energy detector (i.e.,  $N = 1$ ), the theoretical expression for the SER in Rayleigh flat fading is [28]

$$\text{SER} = \frac{3}{\text{SNR} + 2} - \frac{3}{2\text{SNR} + 3} + \frac{1}{3\text{SNR} + 4}. \quad (66)$$

For the complexity plot in Fig. 10, we recall that the cost of the proposed algorithm equals  $J_4(N)$ , given by (37), while the cost of the conventional exhaustive-search approach is  $4^N$ . Once more, we observe that the SER of the noncoherent scheme with  $N = 100$  is nearly equal to the SER of the

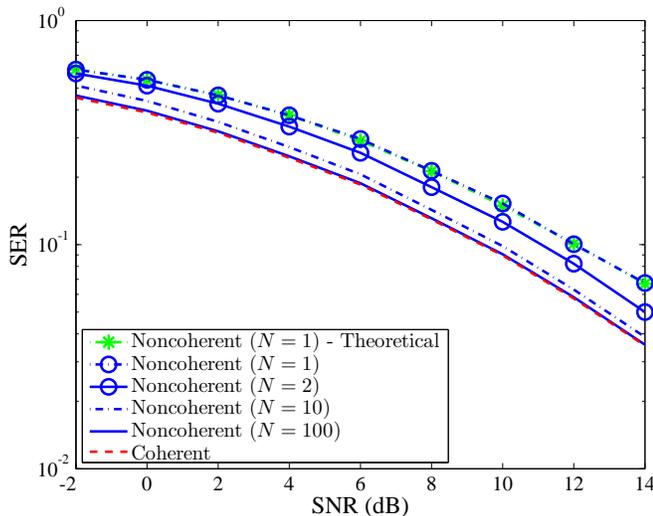


Fig. 9. SER versus symbol SNR of ML/GLRT noncoherent 4FSK detection with sequence length  $N = 1, 2, 10, 100$  and ML coherent 4FSK detection with perfect channel knowledge.

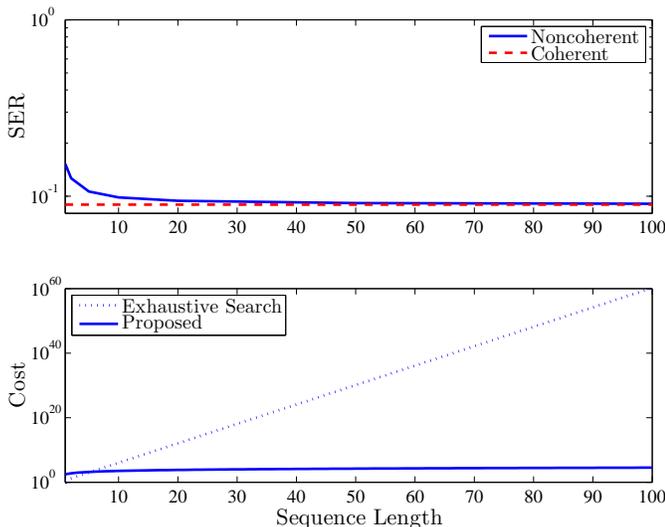


Fig. 10. SER and computational cost of ML/GLRT noncoherent 4FSK detection versus sequence length  $N$  for  $\text{SNR} = 10\text{dB}$ .

coherent one with perfect channel knowledge. Interestingly, this is achieved with  $J_2(100) \simeq 2,360$  operations for BFSK and at most  $J_4(100) \simeq 33,140$  operations for 4FSK (while the conventional exhaustive search is infeasible for such a sequence length since it would require  $2^{100} \simeq 10^{30}$  and  $4^{100} \simeq 10^{60}$ , respectively, operations), opening avenues for practical deployments.

## VI. CONCLUSIONS

For the first time in the literature, this work presented algorithms that perform optimal noncoherent sequence detection with log-linear (in the sequence length) complexity of orthogonally as well as FMO modulated signals over flat fading. We demonstrated that, as the sequence length increases, the proposed detection schemes offer zero BER/SER performance gap compared to ML coherent detection. The above facts

render the adoption of the proposed noncoherent sequence detection schemes for practical deployments as a probable option in the power-limited regime.

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