NONCOHERENT SEQUENCE DETECTION OF ORTHOGONALLY MODULATED SIGNALS IN FLAT FADING WITH LOG-LINEAR COMPLEXITY

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ABSTRACT

Frequency-shift keying (FSK) is an orthogonal modulation technique that is primarily used in relatively low-rate communication systems that operate in the power-limited regime. Optimal noncoherent detection of FSK takes the form of sequence detection and has exponential complexity in the sequence length when implemented through an exhaustive search among all possible sequences. In this work, for the first time in the literature, we present an algorithm that performs generalized-likelihood-ratio-test (GLRT) optimal noncoherent sequence detection of orthogonally modulated signals in flat fading with log-linear complexity in the sequence length. Moreover, for Rayleigh fading channels, the proposed algorithm is equivalent to the maximum-likelihood (ML) noncoherent sequence detector. Finally, we show that our algorithm also solves efficiently the optimal noncoherent sequence detection problem in contemporary radio-frequency-identification (RFID) systems.

1. INTRODUCTION

FSK is an orthogonal modulation technique that is primarily used (or considered for future use) in relatively low-rate communication systems that operate in the power-limited regime. Such systems include underwater communications [1]-[6], power-line communications [7], RFID [8]-[10], and cooperative communications [11]-[14]. The common characteristic of the above applications is the low power at which the system operates, making channel estimation intractable. Instead, noncoherent (or blind) detection is usually preferable for such scenaria [7], [9], [12], [14]. Certainly, due to channel-induced memory, optimal noncoherent detection of FSK takes the form of sequence detection [15]-[23] and offers significant performance gains in comparison with conventional single-symbol noncoherent detection [24], at the cost of exponential complexity in the sequence length [14], [18].

In this work, for the first time in the literature, we present an algorithm that performs optimal noncoherent sequence detection of orthogonally modulated signals in flat fading with log-linear complexity in the sequence length. Specifically, the proposed algorithm performs optimal GLRT sequence detection. Moreover, for Rayleigh fading channels, it is equivalent to the ML noncoherent sequence detector. As a final note, we show that noncoherent sequence detection of FM0 signals[1] is equivalent to noncoherent sequence detection of binary FSK (BFSK). Hence, our algorithm solves efficiently the optimal sequence detection problem in contemporary RFID systems. Our algorithm is based on principles that have been used for polynomial-complexity optimization in [31]-[33] and complements efficient noncoherent detection techniques that have been developed for phase-shift-keying [31] and pulse-amplitude-modulation [33]-[35] signals.

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2. FREQUENCY-SHIFT KEYING

2.1. Signal Model and Optimal Sequence Detection

M-ary FSK (MFSK) utilizes M sub-carrier frequencies to modulate the information symbol $x \in \mathcal{M} \triangleq \{1, 2, \ldots, M\}$.[2] Since it is an orthogonal modulation method, the discrete baseband equivalent received signal for a single symbol duration is written as

$$\mathbf{r} = \sqrt{P} h \mathbf{e}_x + \mathbf{n} \tag{1}$$

where P denotes signal power, h is a complex channel coefficient,[3] $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}_M, \sigma_w^2 \mathbf{I}_M)$ denotes additive Gaussian noise, and $\mathbf{e}_x = [\underbrace{0\ \ldots\ 0}_{x-1}\ \ 1\ \ \underbrace{0\ \ldots\ 0}_{M-x}]^T$ is the xth column of the $M \times M$ identity matrix $\mathbf{I}_M$. For notation simplicity, we also define set $\mathcal{I}_M \triangleq \{\mathbf{e}_1, \ldots, \mathbf{e}_M\}$.

In this work, we consider transmission of an $N \times 1$ symbol sequence $\mathbf{x} = [x_1, \ldots, x_N]^T \in \mathcal{M}^N$. If $\mathbf{y}_1, \ldots, \mathbf{y}_N$ are the corresponding received vectors (per information symbol) given by (1), then we may form the received vector for the

[1] FM0 is a line coding technique that is utilized by the current RFID standards [25]-[30].

[2] We present our developments in the context of FSK signals. However, we note that they hold for any orthogonally modulated signaling technique.

[3] We assume that the channel coefficient is the same over each frequency (flat fading).

entire sequence $\mathbf{x}$ as $\mathbf{y} \triangleq \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \sqrt{P}h \underbrace{\begin{bmatrix} \mathbf{e}_{x_1} \\ \vdots \\ \mathbf{e}_{x_N} \end{bmatrix}}_{\mathbf{s}} + \mathbf{w}$, where $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{MN})$.

We assume that the channel coefficient h is not available. For Rayleigh fading (i.e., $h \sim \mathcal{CN}(0, \sigma_h^2)$), the ML noncoherent detector (which maximizes the conditional probability density function of $\mathbf{y}$ given $\mathbf{s}$) can be shown to take the form[4]

$$\hat{\mathbf{s}}^{\mathrm{ML}} = \arg\max_{\mathbf{s} \in \mathcal{I}_M^N} \left|\mathbf{s}^T \mathbf{y}\right|^2 \tag{2}$$

$$\Leftrightarrow \hat{\mathbf{x}}^{\mathrm{ML}} = \arg\max_{\mathbf{x} \in \mathcal{M}^N} \left|y_1[x_1] + \ldots + y_N[x_N]\right|. \tag{3}$$

For non-Rayleigh or unknown channel distribution, if we consider joint channel estimation and data detection, i.e., GLRT sequence detection [34], then[5]

$$\hat{\mathbf{s}}^{\mathrm{GLRT}} = \arg\min_{\mathbf{s} \in \mathcal{I}_M^N} \left\{ \min_{h \in \mathbb{C}} \left\| \mathbf{y} - \sqrt{P}h\mathbf{s} \right\|^2 \right\} = \arg\max_{\mathbf{s} \in \mathcal{I}_M^N} \left|\mathbf{s}^T \mathbf{y}\right|. \tag{4}$$

Hence, the ML optimization problem in (2) and the GLRT optimization problem in (4) are equivalent. In the following, we present an algorithm that solves the above problems with log-linear complexity $\mathcal{O}(N \log N)$, as opposed to the exponential complexity $\mathcal{O}(M^N)$ of the conventional exhaustive search.

2.2. Log-linear-complexity Optimal Detection

First, we present the proposed algorithm for BFSK ($M = 2$). Then, we generalize to any $M \geq 2$. In either case, we utilize the fact that

$$\begin{aligned} &\max_{\mathbf{x} \in \mathcal{M}^N} \left|y_1[x_1] + \ldots + y_N[x_N]\right| \\ &= \max_{\mathbf{x} \in \mathcal{M}^N} \max_{\phi \in [0, 2\pi)} \Re\left\{ \mathsf{e}^{-j\phi} \left(y_1[x_1] + \ldots + y_N[x_N]\right) \right\} \\ &= \max_{\phi \in [0, 2\pi)} \max_{\mathbf{x} \in \mathcal{M}^N} \left\{ \Re\left\{ \mathsf{e}^{-j\phi} y_1[x_1] \right\} + \ldots + \Re\left\{ \mathsf{e}^{-j\phi} y_N[x_N] \right\} \right\}. \end{aligned} \tag{5}$$

2.2.1. *Optimal algorithm for $M = 2$*

For a given point $\phi \in [0, 2\pi)$, the innermost maximization in (5) is separable for each x_n and, hence, splits into independent maximizations for any $n = 1 \ldots, N$, as

$$\begin{aligned} &\hat{x}_n = \arg\max_{x_n \in \{1,2\}} \Re\left\{ \mathsf{e}^{-j\phi} y_n[x_n] \right\} \\ &\Leftrightarrow \Re\left\{ \mathsf{e}^{-j\phi} y_n[1] \right\} \underset{\hat{x}_n = 2}{\overset{\hat{x}_n = 1}{\gtrless}} \Re\left\{ \mathsf{e}^{-j\phi} y_n[2] \right\} \\ &\Leftrightarrow \cos\left(\phi - \angle y_n[1] - y_n[2]\right) \underset{\hat{x}_n = 2}{\overset{\hat{x}_n = 1}{\gtrless}} 0 \end{aligned} \tag{6}$$

where $\hat{x}_n$ is the decision on the information symbol x_n at the nth time slot and $\angle z$ denotes the angle of the complex number z.

According to (6), as ϕ scans $[0, 2\pi)$, the decision $\hat{x}_n$ changes only when $\phi = \underbrace{\pm\frac{\pi}{2} + \angle y_n[1] - y_n[2] \pmod{2\pi}}_{\phi_n^{(1)}, \phi_n^{(2)}}$.
Hence, the sequence decision $\hat{\mathbf{x}} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N]$ changes only at $\phi_1^{(1)}, \phi_1^{(2)}, \phi_2^{(1)}, \phi_2^{(2)}, \ldots, \phi_N^{(1)}, \phi_N^{(2)}$. If we sort the above phases, i.e., $(\theta_1, \theta_2, \ldots, \theta_{2N}) = \mathsf{sort}(\phi_1^{(1)}, \phi_1^{(2)}, \phi_2^{(1)}, \phi_2^{(2)}, \ldots, \phi_N^{(1)}, \phi_N^{(2)})$, then the decision $\hat{\mathbf{x}}$ remains constant in each one of the $2N$ intervals $C_0 = (0, \theta_1)$, $C_1 = (\theta_1, \theta_2)$, $\ldots$, $C_{2N-1} = (\theta_{2N-1}, \theta_{2N})$. Note that we ignore $(\theta_{2N}, 2\pi)$ because it gives the same sequence $\hat{\mathbf{x}}$ with C_0. Our objective is the identification of the $2N$ sequences $\hat{\mathbf{x}}_0, \hat{\mathbf{x}}_1, \ldots, \hat{\mathbf{x}}_{2N-1}$ (that correspond to the $2N$ intervals $C_0, C_1, \ldots, C_{2N-1}$), one of which is $\hat{\mathbf{x}}^{\mathrm{ML}}$.

We observe that the candidate sequence that we obtain at any $\phi \in [0, \pi)$ is the complement of the candidate sequence that we obtain at $\phi + \pi$.[6] Hence, it suffices to identify the N candidate sequences at $[0, \theta_N)$ and, then, consider also their complements,[7] i.e.,

$$\hat{\mathbf{x}}_{n+N} = \hat{\mathbf{x}}_n^{\mathrm{c}}, \quad n = 0, 1, \ldots, N-1. \tag{7}$$

We also observe that sequences that correspond to adjacent intervals differ in exactly one element. For example, $\hat{\mathbf{x}}_0$ and $\hat{\mathbf{x}}_1$ differ in the element that produced θ_1. Hence, we propose to *(i)* identify $\hat{\mathbf{x}}_0$ at $\phi = 0$ through (6), *(ii)* compute $\hat{\mathbf{x}}_0^{\mathrm{c}}$, and *(iii)* successively visit the angles $\theta_1, \ldots, \theta_{N-1}$ to produce the remaining sequences (and their complements), evaluate their metric in (3), and track the best sequence and its metric. Note that, at each point θ_n, the new sequence $\hat{\mathbf{x}}_n$ is produced by changing only one element of the preceding sequence $\hat{\mathbf{x}}_{n-1}$. The metric of $\hat{\mathbf{x}}_n$ is obtained by simply updating the metric of $\hat{\mathbf{x}}_{n-1}$ with respect to the single element that changed at θ_n.

The pseudo-code of our proposed BFSK noncoherent ML/GLRT sequence detection algorithm is illustrated in Fig. 1. The overall complexity of the proposed algorithm is dominated by the computational cost of phase sorting at line 4 which is on the order of $\mathcal{O}(N \log N)$.

2.2.2. *Optimal algorithm for $M > 2$*

If we fix $\phi \in [0, 2\pi)$, then the innermost maximization in (5) splits into independent maximizations, $\forall\, n = 1, \ldots, N$, as

$$\hat{x}_n = \arg\max_{x \in \mathcal{M}} \Re\left\{ \mathsf{e}^{-j\phi} y_n[x] \right\}. \tag{8}$$

We observe that, for fixed ϕ, (8) is solved by selecting the largest value of $\Re\{\mathsf{e}^{-j\phi} \mathbf{y}_n\}$. As ϕ scans $[0, 2\pi)$, the decision

[4] We use the notation $\mathbf{y}_n = \begin{bmatrix} y_n[1] \\ \vdots \\ y_n[M] \end{bmatrix}$ to represent the $M \times 1$ vector $\mathbf{y}_n$.

[5] We recall that GLRT detection is independent of channel distribution, hence h does not have to be complex Gaussian as in the ML detection in (2).

[6] Since the constellation is binary, we use the term "complementary sequences" to indicate sequences $\mathbf{x}$ and $\mathbf{y}$ that are related by $\mathbf{y}^{\mathrm{c}} = \mathbf{x}$ (i.e., $y_n^{\mathrm{c}} = x_n,\ n = 1, 2, \ldots, N$) where $1^{\mathrm{c}} = 2$ and $2^{\mathrm{c}} = 1$.

[7] Note that we ignore $[\theta_N, \pi)$ because it corresponds to the complementary sequence $\hat{\mathbf{x}}_0^{\mathrm{c}}$.

Algorithm 1 Optimal Noncoherent Binary Orthogonal Detection in Time $\mathcal{O}(N\log N)$

Input: $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N$

for $n = 1 : N$ **do**
    $\phi_n := \frac{\pi}{2} + \angle(y_n[1] - y_n[2]) \pmod{\pi}$
end for
$(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_N) := \mathsf{sort}(\phi_1, \phi_2, \ldots, \phi_N)$
for $n = 1 : N$ **do**
    $\hat{x}_n := \arg\max(\Re\{y_n[1]\}, \Re\{y_n[2]\})$
end for
$value\_\hat{\mathbf{x}} := y_1[\hat{x}_1] + y_2[\hat{x}_2] + \ldots + y_N[\hat{x}_N]$
$value\_\hat{\mathbf{x}}^{c} := y_1[\hat{x}_1^{c}] + y_2[\hat{x}_2^{c}] + \ldots + y_N[\hat{x}_N^{c}]$
$[\mathrm{ML}\_value, \hat{\mathbf{x}}_{\mathrm{best}}] := \max(|value\_\hat{\mathbf{x}}|, |value\_\hat{\mathbf{x}}^{c}|)$
$\hat{\mathbf{x}}^{\mathrm{ML}} := \hat{\mathbf{x}}_{\mathrm{best}}$
for $i = 1 : N-1$ **do**
    let $\mathbf{y}_n$ be the received vector for which $\boldsymbol{\theta}_i$ was obtained
    $value\_\hat{\mathbf{x}} := value\_\hat{\mathbf{x}} - y_n[\hat{x}_n] + y_n[\hat{x}_n^{c}]$
    $value\_\hat{\mathbf{x}}^{c} := value\_\hat{\mathbf{x}}^{c} - y_n[\hat{x}_n^{c}] + y_n[\hat{x}_n]$
    $\hat{x}_n = \hat{x}_n^{c}$
    $[best\_value, \hat{\mathbf{x}}_{\mathrm{best}}] := \max(|value\_\hat{\mathbf{x}}|, |value\_\hat{\mathbf{x}}^{c}|)$
    if $best\_value > \mathrm{ML}\_value$ **then**
        $\mathrm{ML}\_value := best\_value$
        $\hat{\mathbf{x}}^{\mathrm{ML}} := \hat{\mathbf{x}}_{\mathrm{best}}$
    end if
end for

Output: $\hat{\mathbf{x}}^{\mathrm{ML}}$

Fig. 1. ML/GLRT sequence detection algorithm for BFSK and FM0.

$\hat{x}_n$ may change only when, for some $k, l \in \mathcal{M}$ with $k \neq l$, $\Re\{\mathrm{e}^{-j\phi} y_n[k]\} = \Re\{\mathrm{e}^{-j\phi} y_n[l]\} \Leftrightarrow \Re\{\mathrm{e}^{-j\phi}(y_n[k] - y_n[l])\} = 0 \Leftrightarrow \cos\big(\phi - \angle(y_n[k] - y_n[l])\big) = 0$, i.e.,

$$\phi = \underbrace{\pm\frac{\pi}{2} + \angle(y_n[k] - y_n[l]) \pmod{2\pi}}_{\phi^{(1)}_{n,\{k,l\}},\, \phi^{(2)}_{n,\{k,l\}}}. \tag{9}$$

We note that there exist $NM(M-1)$ such ϕ's. However, it turns out that the decision $\hat{\mathbf{x}}$ changes at only (at most) $2MN$ points. This is stated in the following proposition (the proof is omitted due to lack of space).

Proposition 1 *For $M > 2$, there exist at most $2MN$ changes of the sequence decision $\hat{\mathbf{x}}$ in the interval $[0, 2\pi)$.* □

The above proposition states that, for MFSK, it suffices to check at most $2MN$ phases where the sequence decision changes. When the phases have been determined, the remaining process resembles to the algorithm of case $M = 2$. The complete optimal algorithm for MFSK sequence detection is depicted in Fig. 2. The overall complexity of the algorithm is dominated by the sorting operation at line 28 and, thus, the worst-case complexity of the algorithm is $\mathcal{O}(N\log N)$.

Algorithm 2 Optimal Noncoherent M-ary Orthogonal Detection in time $\mathcal{O}(N\log N)$

Input: $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N$

1: **for** $n = 1 : N$ **do**
2:     $\hat{x}_n := \arg\max(\Re\{y_n[1]\}, \Re\{y_n[2]\}, \ldots, \Re\{y_n[M]\})$
3: **end for**
4: $\hat{\mathbf{x}}^{\phi} \triangleq [\hat{x}_1^{\phi}, \hat{x}_2^{\phi}, \ldots, \hat{x}_N^{\phi}] := \hat{\mathbf{x}}$
5: **for** $n = 1 : N$ **do**
6:     **for** $k, l \in \mathcal{M} \times \mathcal{M}$ such that $k \neq l$ **do**
7:         $\phi^{(1)}_{n,\{k,l\}} := +\frac{\pi}{2} + \angle(y_n[k] - y_n[l]) \pmod{2\pi}$
8:         $\phi^{(2)}_{n,\{k,l\}} := -\frac{\pi}{2} + \angle(y_n[k] - y_n[l]) \pmod{2\pi}$
9:     **end for**
10: **end for**
11: **for** $n = 1 : N$ **do**
12:     $\boldsymbol{\phi}_n := \mathsf{sort}\left(\phi^{(1)}_{n,\{1,2\}}, \phi^{(2)}_{n,\{1,2\}}, \phi^{(1)}_{n,\{1,3\}}, \phi^{(2)}_{n,\{1,3\}}, \ldots, \phi^{(1)}_{n,\{M-1,M\}}, \phi^{(2)}_{n,\{M-1,M\}}\right)$
13: **end for**
14: $\boldsymbol{\phi} := []$
15: **for** $n = 1 : N$ **do**
16:     **for** $j = 1 : M(M-1)$ **do**
17:         get the pair $\{k, l\}$ associated with $\phi_n[j]$ from line 12
18:         **if** $\hat{x}_n^{\phi} \in \{k, l\}$ **then**
19:             **if** $\hat{x}_n^{\phi} = k$ **then**
20:                 $\hat{x}_n^{\phi} := l$
21:             **else**
22:                 $\hat{x}_n^{\phi} := k$
23:             **end if**
24:             $\boldsymbol{\phi} := [\boldsymbol{\phi}, \phi_n[j]]$
25:         **end if**
26:     **end for**
27: **end for**
28: $\boldsymbol{\theta} := \mathsf{sort}(\boldsymbol{\phi})$
29: $\hat{\mathbf{x}}^{\mathrm{ML}} := \hat{\mathbf{x}}$
30: $value\_\hat{\mathbf{x}} := y_1[\hat{x}_1] + y_2[\hat{x}_2] + \ldots + y_N[\hat{x}_N]$
31: $\mathrm{ML}\_value := |value\_\hat{\mathbf{x}}|$
32: **for** $i = 1 : \mathsf{length}(\boldsymbol{\theta})$ **do**
33:     get the pair $\{k, l\}$ associated with $\theta_i = \phi^{(1)}_{n,\{k,l\}}$ or $\theta_i = \phi^{(2)}_{n,\{k,l\}}$
34:     **if** $\hat{x}_n = k$ **then**
35:         $value\_\hat{\mathbf{x}} = value\_\hat{\mathbf{x}} - y_n[k] + y_n[l]$
36:         $\hat{x}_n := l$
37:     **else**
38:         $value\_\hat{\mathbf{x}} = value\_\hat{\mathbf{x}} - y_n[l] + y_n[k]$
39:         $\hat{x}_n := k$
40:     **end if**
41:     $best\_value := |value\_\hat{\mathbf{x}}|$
42:     **if** $best\_value > \mathrm{ML}\_value$ **then**
43:         $\mathrm{ML}\_value := best\_value$
44:         $\hat{\mathbf{x}}^{\mathrm{ML}} := \hat{\mathbf{x}}$
45:     **end if**
46: **end for**

Output: $\hat{\mathbf{x}}^{\mathrm{ML}}$

Fig. 2. ML/GLRT sequence detection algorithm for MFSK.

3. FM0 LINE CODING

FM0 is a line-coding technique that is used in the current RFID communications standard. In FM0, the signal level takes two possible values; namely, 1 and 0. Specifically, it changes at the middle of the bit period for bit "$\emptyset$," whereas for bit "1" the level remains constant. Moreover, it always changes at the bit boundaries, as can been seen in Fig. 3, and, thus, the signals from one bit interval to another are not independent (i.e., FM0 induces memory).

We assume a sequence of N information bits which, for convenience, are represented in the "logical form," i.e., $b_1, b_2, \ldots, b_N \in \{\emptyset, 1\}$, where $\emptyset^{c} = 1$ and $1^{c} = \emptyset$. If we denote by $d_n \in \{\emptyset, 1\}$ the signal level at the end of the nth bit period, then $d_n = d_{n-1} \oplus b_n$, $n = 1, 2, \ldots, N$. Hence, during the nth bit period, the signal level takes the values $(d_{n-1}^{c}, \underbrace{d_{n-1} \oplus b_n}_{d_n})$. As a result, the transmitted sequence that corresponds to the information sequence $b_1, b_2, \ldots, b_N$ is

$$d_0 \left| d_0^{c}, \underbrace{d_0 \oplus b_1}_{d_1} \right| d_1^{c}, \underbrace{d_1 \oplus b_2}_{d_2} \left| \ldots \right| d_{N-1}^{c}, \underbrace{d_{N-1} \oplus b_N}_{d_N} \Big| d_N^{c} \tag{10}$$

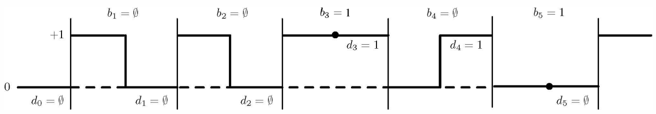

Fig. 3. FM0 transmit waveform.

or, in vector form, $[d_0, d_0^c, d_1, d_1^c, \dots, d_N, d_N^c]^T = \underbrace{\begin{bmatrix} \mathbf{e}_{d_0} \\ \vdots \\ \mathbf{e}_{d_N} \end{bmatrix}}_{\mathbf{d}}$, where $d_0 \in \{\emptyset, 1\}, d_n = d_{n-1} \oplus b_n = d_0 \oplus b_1 \oplus b_2 \oplus \dots \oplus b_n$, $n = 1, 2, \dots, N, \mathbf{e}_\emptyset = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right], \mathbf{e}_1 = \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$, and $\oplus$ denotes exclusive-OR operation. Upon transmission over a flat-fading channel, the received vector is $\mathbf{y} = \begin{bmatrix} \mathbf{y}_0 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \sqrt{P} h \mathbf{d} + \mathbf{w}$ where P is the signal power, h is a complex channel coefficient, and $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_{2(N+1)})$.

The optimal noncoherent FM0 sequence detector becomes $\hat{\mathbf{d}}^{\mathrm{ML}} = \arg\max_{\mathbf{d} \in \mathcal{I}_2^{N+1}} |\mathbf{d}^T \mathbf{y}|$. The GLRT detector can be shown to admit the same decision rule. Hence, the algorithm of Fig. 1 for BFSK can be directly employed to the received vector $\mathbf{y}$ to obtain the sequence $\hat{\mathbf{d}}^{\mathrm{ML}}$ with complexity $\mathcal{O}(N \log N)$. Then, the optimal information sequence $\hat{\mathbf{b}}^{\mathrm{ML}}$ is obtained by $\hat{b}_n^{\mathrm{ML}} = \hat{d}_n^{\mathrm{ML}} \oplus \hat{d}_{n-1}^{\mathrm{ML}}$, $n = 1, 2, \dots, N$.

4. SIMULATION RESULTS

We consider BFSK transmissions through a Rayleigh flat fading channel with $\sigma_h^2 = 1$. In Fig. 4, we plot the bit error rate (BER) of the optimal noncoherent sequence detector as a function of the transmitted signal-to-noise ratio (SNR), for sequence length $N = 1, 2, 10, 100$. We include the BER of the conventional ML coherent detector, as a reference. We observe that, as the sequence length increases, the noncoherent detector approaches the coherent one in terms of BER. Moreover, the BER of the conventional noncoherent detector (i.e., $N = 1$) is 3dB far from the coherent one; as the sequence length N increases, the BER gap decreases to zero.

To demonstrate the rate of convergence to coherent detection performance, in Fig. 5, we set the SNR to 10dB and plot the BER of the optimal noncoherent detector and the computational cost of the proposed algorithm and the conventional exhaustive-search approach as a function of the sequence length N. Finally, in Fig. 6, we repeat the above study for 4FSK modulation and make similar observations for its symbol error rate (SER).

We note that the BER/SER of the noncoherent scheme with $N = 100$ is nearly equal to the BER/SER of the coherent one with perfect channel knowledge. Interestingly, this is achieved with complexity on the order of $100\log_2 100 \simeq 700$ computations (while the conventional exhaustive search would require 2^{100} or 4^{100} computations for BFSK or 4FSK, respectively), opening avenues for practical deployments.

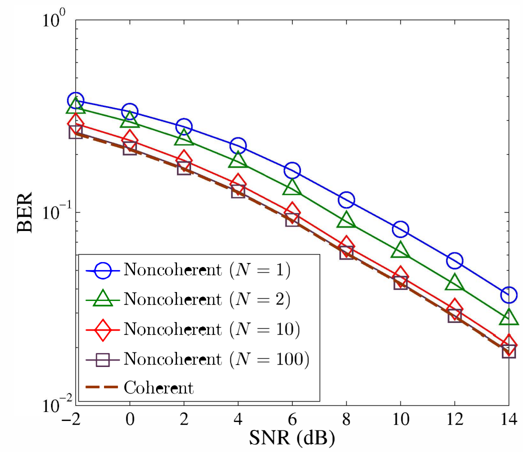

Fig. 4. BFSK BER versus bit SNR for ML/GLRT noncoherent detection with sequence length $N = 1, 2, 10, 100$ and ML coherent detection.

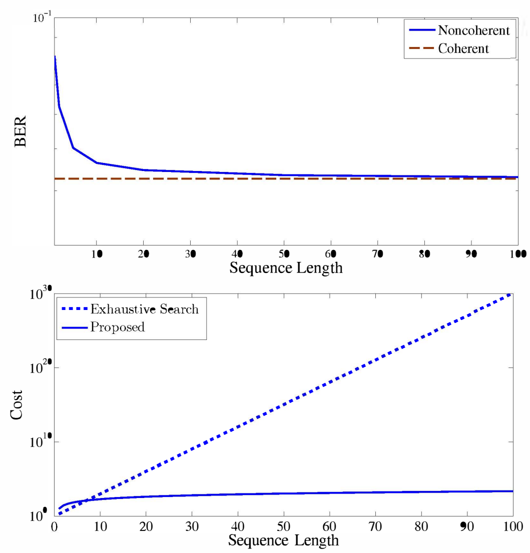

Fig. 5. BFSK BER and cost versus sequence length (SNR=10dB).

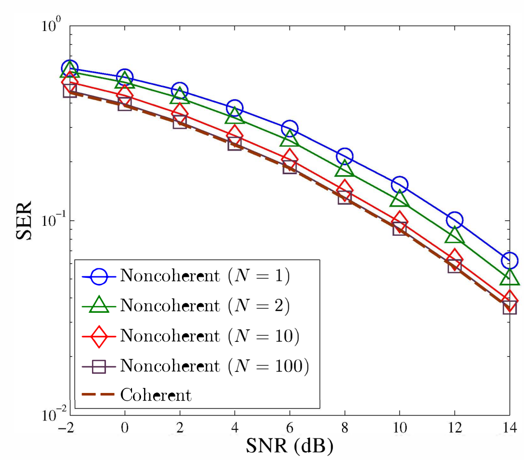

Fig. 6. 4FSK SER versus symbol SNR for ML/GLRT noncoherent detection with sequence length $N = 1, 2, 10, 100$ and ML coherent detection.

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