

# Multiuser Differential-PSK Demodulators for DS/CDMA Signals

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## ABSTRACT

Second-order multipath channel estimation procedures for direct-sequence code-division-multiple-access (DS/CDMA) communications induce phase ambiguity that necessitates differential phase-shift-keying (DPSK) modulation and detection. The maximum likelihood (ML) single-symbol multiuser DPSK/CDMA detector is derived with direct generalization to multiple-symbol (block) multiuser DPSK/CDMA detection. Exponential complexity requirements limit the use of the ML rule to theoretical lower-bound bit-error-rate benchmarking. Linear filter DPSK demodulators are viewed as a practical alternative. Phase-ambiguous RAKE filtering followed by RAKE-output differential detection is considered. The familiar minimum-variance-distortionless-response (MVDR) PSK/CDMA filter (designed for minimum *filter output* energy under the constraint of distortionless response in a given RAKE vector direction) adds the valuable feature of active interference suppression; however, minimum disturbance variance at the *differential logic output* can be claimed formally only in the absence of multipath (no inter-symbol-interference). Short-data-record adaptive alternatives to costly and slow adaptive MVDR implementations are sought in the context of auxiliary-vector (AV) filtering. Numerical and simulation studies illustrate the developments.

**Keywords:** Adaptive filters, code division multiaccess, differential phase shift keying, interference suppression, maximum likelihood detection, multipath channels.

## 1. INTRODUCTION

Differential modulation and detection is preferred in phase-shift-keying transmission systems when effective estimation of the channel induced phase is not feasible.<sup>1,2</sup> The unknown “channel phase” may be due to the transmission channel itself (fading phenomena) or due to the receiver part of the system (asynchronous carrier demodulation). In both cases, successful estimation of the phase is required when coherent detection is chosen for the recovery of the transmitted information symbols. In realistic situations, however, channel phase estimation is a non-trivial task, especially when the channel is time-varying and some form of phase tracking is needed. Considered as a non-coherent communication technique, differential phase-shift keying (DPSK) does not require explicit knowledge of the channel phase and provides us with a valuable alternative to coherent detection methods.

The penalty paid for the use of non-coherent DPSK demodulation is an arguably reasonable performance loss in comparison with coherent PSK. For example, the performance loss versus coherent BPSK for the conventional 1-lag Binary DPSK demodulator is less than  $2dB$  at the  $10^{-2}$  Bit-Error-Rate level and about  $1dB$  at the  $10^{-4}$  BER level.<sup>1</sup> The conventional 1-lag, or “single-symbol”, DPSK demodulator was generalized under the Maximum Likelihood criterion to multiple-symbol (or “block”) detection procedures.<sup>3</sup> While ML block DPSK detection recovers a significant portion of the performance loss in comparison with ideal coherent PSK, this comes at the cost of exponentially increased computational complexity. Moreover, multiple-symbol detection techniques assume a constant channel phase for the whole transmission period corresponding to the block of symbols to be detected. In contrast, 1-lag DPSK detection assumes that the channel phase remains constant during only two successively transmitted symbols.

In general communications systems, multipath fading phenomena are usually modeled as a convolution operation with random, complex Gaussian, channel coefficients that induce an independent multiplicative disturbance per path

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at the baseband received signal. In this context, the conventional matched filter receiver takes the familiar RAKE form.<sup>1,4</sup> For DS/CDMA communications, which is of particular interest to this present work, the RAKE receiver is simply the superposition of the appropriately shifted versions of the signature waveform of the user of interest multiplied by the corresponding channel coefficients.<sup>5</sup> Since the coefficients are assumed random, the RAKE receiver has to be estimated adaptively, in either a blind or supervised fashion. Second order criteria for blind estimation of the channel coefficients (or equivalently of the RAKE receiver) introduce a global phase ambiguity to the vector coefficient estimate.<sup>6</sup> However, the phase ambiguity problem can be resolved effectively by differential modulation of the information symbol sequence and the use of the output of the RAKE filter at time  $i$  as the reference of its output at time  $i + 1$  (binary or M-ary DPSK modulation and RAKE-output differential detection).

In this present work we consider a general DPSK DS/CDMA communications system and we derive the ML optimum 1-lag (single-symbol) receiver, with straightforward generalization to block (multi-symbol) ML processing. In view of the exponential computational complexity in the number of CDMA users and in terms of the block size, we consider linear filter alternatives for MAI suppression. We begin with RAKE filtering followed by RAKE-output differential detection. This is in sharp contrast to path-by-path differential detection considered in Ref. 1,7,8, which fails to utilize the cross-path differential information present when all channel coefficients are known within the same phase ambiguity.<sup>6</sup> We prove that in the absence of multipath (no Inter-Symbol-Interference) the familiar linear Minimum-Variance-Distortionless-Response (MVDR) PSK/CDMA filter<sup>9</sup> minimizes the disturbance variance at the output of the differential logic (pre-detection variance). Theoretically, when ISI is present the conventional MVDR filter does not qualify as a minimum pre-detection variance linear filter. Yet, we establish that for all practical purposes the MVDR filter and the hard-to-obtain minimum-predetection-variance filter exhibit similar MAI suppression capabilities. For small-sample-support adaptive MAI suppression, an issue of great importance for time-varying channels, we consider Auxiliary-Vector (AV) alternatives<sup>5,10-13</sup> to MVDR DPSK/CDMA receivers. We illustrate the effectiveness of the AV approach in simulated realistic situations (DPSK/CDMA communications over multipath Rayleigh fading AWGN channels).

## 2. SIGNAL MODEL

We consider simultaneous in time and frequency mobile-to-base-station transmissions of  $K$  DS/CDMA users with processing gain  $L$  and DPSK modulation. Each user transmitted signal is assumed to experience multipath Rayleigh fading and the total received signal is the superposition of the multipath faded signals from all users and additive white Gaussian noise. Basic formulation and notation is given below.

The baseband signal transmitted by the  $k$ -th user is represented by

$$u_k(t) = \sum_{i=0}^{\infty} d_k(i) \sqrt{E_k} s_k(t - iT) \quad (1)$$

where  $d_k(i) \in \{-1, 1\}$  is the  $i$ -th transmitted bit,  $E_k$  denotes energy, and  $s_k(t)$  is the assumed normalized user signature waveform

$$s_k(t) = \sum_{l=0}^{L-1} \mathbf{S}_k[l] \psi(t - lT_c). \quad (2)$$

With respect to  $s_k(t)$ ,  $\mathbf{S}_k[l] \in \{-1, 1\}$  is the  $l$ -th bit of the spreading code  $\mathbf{S}_k$  of the  $k$ -th user,  $\psi(t)$  is the chip waveform,  $L$  is the spreading processing gain, and  $T_c = \frac{T}{L}$  is the chip period where  $T$  denotes the  $d_k(i)$  symbol period. We recall that for binary differential phase-shift keying (DPSK), the  $\{b_k(i)\}_{i=1}^{\infty}$  information bit sequence of the  $k$ -th user is differentially encoded and transmitted in the form of the bit sequence  $\{d_k(i)\}_{i=0}^{\infty}$  where

$$d_k(i) = d_k(i-1)b_k(i), \quad d_k(0) = +1. \quad (3)$$

The total baseband signal received at the base station is modeled as follows:

$$r(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \alpha_{k,n} u_k(t - nT_c) e^{j\phi_k} + n(t) \quad (4)$$

where, without loss of generality and for notational simplicity, we assume synchronous signal transmissions and multipath delay spreads that are multiples of the chip period. In (4),  $N$  represents the number of resolvable multipaths

(assumed to be the same for all users) and  $\alpha_{k,n}$ ,  $k = 0, 1, \dots, K-1$ ,  $n = 0, 1, \dots, N-1$ , are independent zero-mean complex Gaussian random variables that model the fading phenomena and are assumed to remain constant over several bit intervals. Carrier phase residuals and other receiver induced phase impairments are absorbed in the channel coefficients  $\alpha_{k,n}$ , while the *unknown* phase  $\phi_k$  represents pre-emptively the phase ambiguity of the channel estimate for user  $k$ ,  $k = 0, 1, \dots, K-1$ , due to second-order channel estimation procedures.<sup>6</sup> The channel additive noise is modeled as a zero-mean complex Gaussian random process  $n(t)$ . After direct sampling of  $r(t)$  at the chip rate  $\frac{1}{T_c}$  (or chip-matched filtering and sampling at the chip rate) we form the  $(L+N-1) \times 1$   $m$ -th received data vector corresponding to the  $m$ -th *transmitted* bit

$$\mathbf{r}_m = [r(mT), r(mT+T_c), \dots, r(mT+(L+N-2)T_c)]^T \quad (5)$$

where  $T$  denotes the transpose operation.

If we define the *effective signature*  $\mathbf{g}_{k,n}$  of the  $k$ -th user with respect to its  $n$ -th path as the  $(L+N-1) \times 1$ ,  $n$ -right-shifted,  $\theta$ -padded version of the original signature vector

$$\mathbf{g}_{k,n}[l] \triangleq \begin{cases} \alpha_{k,n} \mathbf{S}_k[l-n], & l = n, n+1, \dots, L+n-1 \\ 0, & \text{otherwise} \end{cases}, \quad (6)$$

then the  $l$ -th element of the data vector  $\mathbf{r}_m$ ,  $l = 0, 1, \dots, L+N-2$ , can be written as follows:

$$\begin{aligned} \mathbf{r}_m[l] &= \sum_{k=0}^{K-1} d_k(m-1) e^{j\phi_k} \sqrt{E_k} \sum_{n=0}^{N-1} \mathbf{g}_{k,n}[l+L] + \sum_{k=0}^{K-1} d_k(m) e^{j\phi_k} \sqrt{E_k} \sum_{n=0}^{N-1} \mathbf{g}_{k,n}[l] + \\ &+ \sum_{k=0}^{K-1} d_k(m+1) e^{j\phi_k} \sqrt{E_k} \sum_{n=0}^{N-1} \mathbf{g}_{k,n}[l-L] + n_{m,l} \quad (\text{if } N < L) \end{aligned} \quad (7)$$

where  $n_{m,l}$  is a zero-mean complex Gaussian random variable with variance  $N_0$ , and  $n_{m_1,l_1}$ ,  $n_{m_2,l_2}$  are independent and identically distributed (i.i.d.) for  $m_1T + l_1T_c \neq m_2T + l_2T_c$  ( $0 \leq l_1, l_2 \leq L+N-2$ ).

For the sake of notational simplicity and compactness, we may rewrite (7) in vector form as follows:

$$\mathbf{r}_m = \mathbf{M}(\mathbf{d}(m-1), \mathbf{d}(m), \mathbf{d}(m+1), \vec{\phi}) + \mathbf{n}_m \quad (8)$$

where  $\mathbf{d}(m) \triangleq [d_0(m), d_1(m), \dots, d_{K-1}(m)]^T$  represents the current set of differentially encoded symbols,  $\mathbf{d}(m-1) \triangleq [d_0(m-1), d_1(m-1), \dots, d_{K-1}(m-1)]^T$  and  $\mathbf{d}(m+1) \triangleq [d_0(m+1), d_1(m+1), \dots, d_{K-1}(m+1)]^T$  are the previous and the next set of differentially encoded symbols (inducing Inter-Symbol-Interference),  $\vec{\phi} \triangleq [\phi_0, \phi_1, \dots, \phi_{K-1}]^T$  is the vector of unknown phases,  $\mathbf{n}_m \triangleq [n_{m,0}, n_{m,1}, \dots, n_{m,L+N-2}]^T$ , and the  $(L+N-1) \times 1$   $\mathbf{M}(\mathbf{d}(m-1), \mathbf{d}(m), \mathbf{d}(m+1), \vec{\phi})$  vector is defined as follows:

$$\begin{aligned} \mathbf{M}(\mathbf{d}(m-1), \mathbf{d}(m), \mathbf{d}(m+1), \vec{\phi})[l] &\triangleq \sum_{k=0}^{K-1} d_k(m-1) e^{j\phi_k} \sqrt{E_k} \sum_{n=0}^{N-1} \mathbf{g}_{k,n}[l+L] + \\ &+ \sum_{k=0}^{K-1} d_k(m) e^{j\phi_k} \sqrt{E_k} \sum_{n=0}^{N-1} \mathbf{g}_{k,n}[l] + \sum_{k=0}^{K-1} d_k(m+1) e^{j\phi_k} \sqrt{E_k} \sum_{n=0}^{N-1} \mathbf{g}_{k,n}[l-L], \\ &l = 0, 1, \dots, L+N-2. \end{aligned} \quad (9)$$

In the following section we derive the ML optimum receiver for this DPSK/CDMA signal model. As expected, computational complexity considerations limit its potential use to lower-bound BER benchmarking.

### 3. THE MAXIMUM LIKELIHOOD DPSK/CDMA RECEIVER

In this section we focus on the derivation of the maximum likelihood (ML) multiuser 1-lag (single-symbol) detector for DPSK-modulated CDMA communications over Rayleigh fading AWGN channels. The derivation can be generalized to  $M$ -lag block detection in a straightforward manner. Single-symbol multiuser DPSK detection requires processing of a window of the received data stream that includes *two* sequentially transmitted differential bits. During this  $2T + (N - 1)T_c$  time interval we assume that the phase ambiguity per user signal,  $\phi_k$  in (7), due to channel/RAKE estimation procedures, remains constant.

For the detection of the  $m$ -th *information* bit of each user we utilize the received vectors  $\mathbf{r}_{m-1}$  and  $\mathbf{r}_m$ . We note that  $\mathbf{r}_m[l] = \mathbf{r}_{m-1}[l + L]$  for  $l = 0, 1, \dots, N - 2$  and for simplicity in notation we form the  $(2L + N - 1) \times 1$  *combined 2-symbol received vector*  $\mathbf{q}_m$  as follows

$$\mathbf{q}_m[l] \triangleq \begin{cases} \mathbf{r}_{m-1}[l], & l = 0, 1, \dots, L - 1 \\ \mathbf{r}_m[l - L], & l = L, L + 1, \dots, 2L + N - 2. \end{cases} \quad (10)$$

In a similar way, we define the  $(2L + N - 1) \times 1$  *combined signal vector*  $\mathbf{M}_2(\mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m), \mathbf{d}(m+1), \vec{\phi})$ :

$$\mathbf{M}_2(\mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m), \mathbf{d}(m+1), \vec{\phi})[l] \triangleq \begin{cases} \mathbf{M}(\mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m), \vec{\phi})[l], & l = 0, 1, \dots, L - 1 \\ \mathbf{M}(\mathbf{d}(m-1), \mathbf{d}(m), \mathbf{d}(m+1), \vec{\phi})[l - L], & l = L, L + 1, \dots, 2L + N - 2. \end{cases} \quad (11)$$

If  $\mathbf{b}(m) \triangleq [b_0(m), b_1(m), \dots, b_{K-1}(m)]^T$  is the vector with the  $m$ -th *information* bits of all users, then the ML detection rule is<sup>14</sup>

$$\hat{\mathbf{b}}(m) = \arg \max_{\mathbf{b}(m)} \{p_{\mathbf{r}_{m-1}, \mathbf{r}_m / \mathbf{b}(m)}(\mathbf{r}_{m-1}, \mathbf{r}_m)\} = \arg \max_{\mathbf{b}(m)} \{p_{\mathbf{q}_m / \mathbf{b}(m)}(\mathbf{q}_m)\} \quad (12)$$

where  $p_{\mathbf{r}_{m-1}, \mathbf{r}_m / \mathbf{b}(m)}(\cdot)$  denotes the joint probability density function (pdf) of the two received vectors  $\mathbf{r}_{m-1}$ ,  $\mathbf{r}_m$  given the  $m$ -th information bit vector  $\mathbf{b}(m)$  and  $p_{\mathbf{q}_m / \mathbf{b}(m)}(\cdot)$  denotes the pdf of the *combined 2-symbol received vector*  $\mathbf{q}_m$  given  $\mathbf{b}(m)$ . We observe that

$$p_{\mathbf{q}_m / \mathbf{b}(m)}(\cdot) = \frac{1}{2^K} \sum_{\mathbf{d}(m-2)} p_{\mathbf{q}_m / \mathbf{d}(m-2), \mathbf{b}(m)}(\cdot) \quad (13)$$

since  $\mathbf{d}(m-2)$  and  $\mathbf{b}(m)$  are statistically independent. The  $\mathbf{d}(m-2)$  vector that indices the summation operation takes all  $2^K$  possible values and the constant  $\frac{1}{2^K}$  comes from the fact that the transmitted differential bits are independent uniformly distributed binary random variables across all users. Similarly,  $\mathbf{d}(m-2)$ ,  $\mathbf{b}(m-1)$ ,  $\mathbf{b}(m)$ ,  $\mathbf{b}(m+1)$ , and  $\vec{\phi}$  are also statistically independent which implies that

$$p_{\mathbf{q}_m / \mathbf{b}(m)}(\cdot) = \frac{1}{2^{3K}} \sum_{\mathbf{d}(m-2)} \sum_{\mathbf{b}(m-1)} \sum_{\mathbf{b}(m+1)} \frac{1}{(2\pi)^K} \int_{\vec{\phi}} p_{\mathbf{q}_m / \mathbf{d}(m-2), \mathbf{b}(m-1), \mathbf{b}(m), \mathbf{b}(m+1), \vec{\phi}}(\cdot) d\vec{\phi} \quad (14)$$

where the integral is defined over  $[-\pi, \pi]$  for each  $\phi_k$ ,  $k = 0, 1, \dots, K - 1$ , since we treat the unknown user phases  $\phi_0, \phi_1, \dots, \phi_{K-1}$ , as random, independent, and uniformly distributed on  $[-\pi, \pi]$ . As a result, due to (3), (8), and (10), we may rewrite (14) as follows:

$$p_{\mathbf{q}_m / \mathbf{b}(m)}(\cdot) = \frac{1}{(16\pi)^K} \sum_{\mathbf{d}(m-2)} \sum_{\mathbf{d}(m-1)} \sum_{\mathbf{d}(m+1)} \int_{\vec{\phi}} p_{\mathbf{q}_m / \mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m) = \mathbf{d}(m-1) \odot \mathbf{b}(m), \mathbf{d}(m+1), \vec{\phi}}(\cdot) d\vec{\phi} \quad (15)$$

where  $\odot$  denotes element-by-element multiplication of two vectors with the same length.

Accounting for the white Gaussian noise in (8) we obtain

$$\begin{aligned} & p_{\mathbf{q}_m / \mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m) = \mathbf{d}(m-1) \odot \mathbf{b}(m), \mathbf{d}(m+1), \vec{\phi}}(\mathbf{q}_m) = \\ & = \left( \frac{1}{\pi N_0} \right)^{2L+N-1} \exp \left( - \frac{\left\| \mathbf{q}_m - \mathbf{M}_2(\mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m-1) \odot \mathbf{b}(m), \mathbf{d}(m+1), \vec{\phi}) \right\|^2}{N_0} \right). \end{aligned} \quad (16)$$

Substituting (15) and (16) in (12), we derive the ML decision rule

$$\hat{\mathbf{b}}(m) = \arg \max_{\mathbf{b}(m)} \left\{ \sum_{\mathbf{d}(m-2)} \sum_{\mathbf{d}(m-1)} \sum_{\mathbf{d}(m+1)} \int_{\vec{\phi}} \exp \left( -\frac{1}{N_0} \left\| \mathbf{q}_m - \mathbf{M}_2 \left( \mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m-1) \odot \mathbf{b}(m), \mathbf{d}(m+1), \vec{\phi} \right) \right\|^2 \right) d\vec{\phi} \right\}. \quad (17)$$

For channel coefficient vectors  $[\alpha_{k,0}, \alpha_{k,1}, \dots, \alpha_{k,N-1}]^T$  known within an overall phase ambiguity  $\phi_k$  per user,<sup>6</sup> the above equation is the *ML optimum multiuser 1-lag DPSK/CDMA detector*. Under the assumption of *common* uniformly distributed phase ambiguity across all users,  $\phi_0 = \phi_1 = \dots = \phi_{K-1} = \phi$  (second-order combined multi-channel estimation), the ML rule in (17) becomes

$$\hat{\mathbf{b}}(m) = \arg \max_{\mathbf{b}(m)} \left\{ \sum_{\mathbf{d}(m-2)} \sum_{\mathbf{d}(m-1)} \sum_{\mathbf{d}(m+1)} \exp \left( -\frac{1}{N_0} \left\| \mathbf{M}_2 \left( \mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m-1) \odot \mathbf{b}(m), \mathbf{d}(m+1), \mathbf{0} \right) \right\|^2 \right) I_0 \left( \frac{2}{N_0} \left| \mathbf{q}_m^H \mathbf{M}_2 \left( \mathbf{d}(m-2), \mathbf{d}(m-1), \mathbf{d}(m-1) \odot \mathbf{b}(m), \mathbf{d}(m+1), \mathbf{0} \right) \right| \right) \right\} \quad (18)$$

where  $\mathbf{0}$  is the  $K \times 1$  zero vector and  $I_0(v)$ ,  $v \in \mathcal{R}^+$ , is the  $\theta$ -order modified Bessel function of the first kind<sup>14</sup> :

$$I_0(v) \triangleq \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{v \cos \phi} d\phi \simeq \begin{cases} 1 + \frac{v^2}{4}, & v \ll 1 \\ \frac{e^v}{\sqrt{2\pi v}}, & v \gg 1 \end{cases}. \quad (19)$$

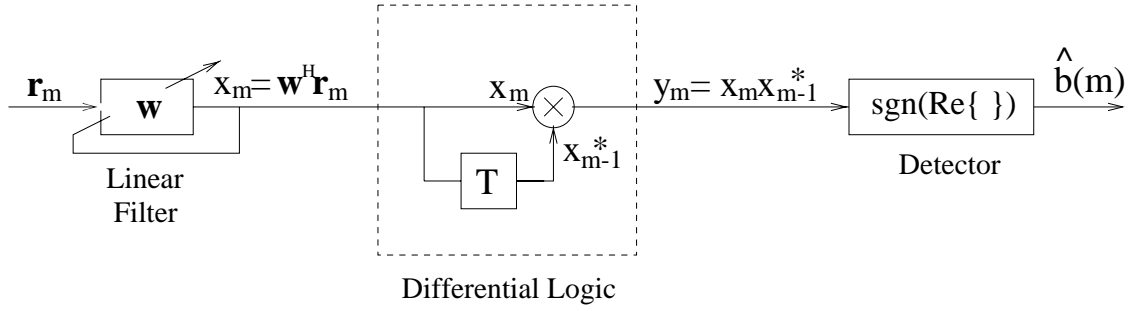
If no reasonable estimate of the channel coefficients  $[\alpha_{k,0}, \alpha_{k,1}, \dots, \alpha_{k,N-1}]^T$ ,  $k = 0, 1, \dots, K-1$ , can be assumed available, then integration of (17) with respect to the probability measure of the complex Gaussian channel coefficients leads to the universal, channel independent, ML decision rule across all Rayleigh fading channel realizations for all users.

#### 4. LINEAR PRE-FILTER DPSK/CDMA RECEIVERS

The multiuser single-symbol ML-optimum DPSK/CDMA detector derived and discussed in the previous section comes with computational complexity that is exponential in the number of users. When generalized to multiple-symbol block detection, the complexity is exponential in the block length, too. This behavior was certainly expected in knowledge of the exponential complexity in the number of users of the optimal PSK/CDMA detector<sup>15</sup> and the exponential complexity in block length of the multiple-symbol ML DPSK demodulator.<sup>3</sup> In search of practical alternatives, in this section we consider single-user single-symbol DPSK/CDMA demodulators that incorporate MAI suppression in the form of linear filtering.

The general structure of a single-user single-symbol DPSK/CDMA receiver with linear pre-filtering is shown in Fig. 1. The received data vector  $\mathbf{r}_m$  is passed through the linear filter  $\mathbf{w}$  whose output is  $x_m = \mathbf{w}^H \mathbf{r}_m$ , where  $^H$  denotes Hermitian transpose. The complex scalar  $x_m$  is delayed by  $T$  (transmitted differential bit period) and conjugated. Then, the output of the differential logic module is the projection of  $x_m$  onto  $x_{m-1}$ , that is  $y_m = x_m x_{m-1}^*$  where  $*$  denotes complex conjugation. Finally, the real part of  $y_m$  is passed through a sign operator and the result is the detected information bit  $\hat{b}(m)$ . The overall DPSK/CDMA demodulator of Fig. 1 maintains filter-phase independent performance. That is, if  $\mathbf{w}^{(2)} = \mathbf{w}^{(1)} e^{j\phi}$  then  $y_m^{(2)} = x_m^{(2)} x_{m-1}^{(2)*} = \left( \mathbf{w}^{(2)H} \mathbf{r}_m \right) \left( \mathbf{w}^{(2)H} \mathbf{r}_{m-1} \right)^* = \left( \mathbf{w}^{(1)H} e^{-j\phi} \mathbf{r}_m \right) \left( \mathbf{w}^{(1)H} e^{-j\phi} \mathbf{r}_{m-1} \right)^* = \left( \mathbf{w}^{(1)H} \mathbf{r}_m \right) \left( \mathbf{w}^{(1)H} \mathbf{r}_{m-1} \right)^* = x_m^{(1)} x_{m-1}^{(1)*} = y_m^{(1)}$  and the final decision statistic is the same regardless of the phase value  $\phi$ . This is the basic characteristic and advantage of DPSK in comparison with coherent demodulation techniques.

The conventional RAKE filter can be viewed as a candidate for the linear filter block  $\mathbf{w}$  in the demodulator structure of Fig. 1. We recall that the RAKE filter is formally the weighted sum of shifted versions of the signature



**Figure 1.** Single-user single-symbol DPSK/CDMA receiver with adaptive linear pre-filtering.

of the user of interest where the weights are the channel coefficients of the paths that correspond to the signature shifts. As a result, according to the signal model in (2) and (4), the length of the RAKE vector is  $L + N - 1$ . Without loss of generality, let us assume that User 0 is the user of interest. Then, the corresponding RAKE filter  $\mathbf{V}_0$  is given by

$$\mathbf{V}_0[l] = \sum_{n=0}^{N-1} \mathbf{g}_{0,n}[l], \quad l = 0, 1, \dots, L + N - 2, \quad (20)$$

where  $\mathbf{g}_{0,n}$ ,  $n = 0, 1, \dots, N - 1$ , is defined in (6). Exact knowledge of the channel coefficients  $\alpha_{0,0}, \alpha_{0,1}, \dots, \alpha_{0,N-1}$  and the signature code of User 0  $\mathbf{S}_0[l]$ ,  $l = 0, 1, \dots, L - 1$ , completely determines the corresponding RAKE filter. However, as previously mentioned, the RAKE-DPSK demodulator of the form of Fig. 1 is insensitive to global phase ambiguities of the channel-vector/RAKE-filter estimate that second-order channel estimation algorithms induce.

RAKE multipath combining with  $\mathbf{V}_0$  is an “effective signature” matched-filter type of operation that maintains Mean-Square (MS) optimality for antipodal single-user transmissions in AWGN or for multiuser transmissions as long as  $\mathbf{V}_0 \perp \mathbf{V}_k$ ,  $k = 1, 2, \dots, K - 1$ , (Ref. 5). Since the latter condition is not met in realistic DS/CDMA communication systems, MS optimality considerations lead to the familiar Minimum-Variance-Distortionless-Response (MVDR) filter, which was considered extensively for BPSK/CDMA systems.<sup>9</sup> MVDR processing minimizes the disturbance (MAI plus noise) variance *at the filter output* while the received signal component in the  $\mathbf{V}_0$  direction is maintained. In other words,  $\mathbf{w}_{\text{MVDR}} \triangleq \arg \min_{\mathbf{w}} \left\{ E \left\{ |x_m|^2 \right\} \right\}$  subject to the constraint  $\mathbf{w}^H \mathbf{V}_0 = 1$ . The solution to this constraint optimization problem is well known:

$$\mathbf{w}_{\text{MVDR}} = \frac{R^{-1} \mathbf{V}_0}{\mathbf{V}_0^H R^{-1} \mathbf{V}_0} \quad (21)$$

where  $R = E \{ \mathbf{r}_m \mathbf{r}_m^H \}$  is the autocorrelation matrix of the received data vector  $\mathbf{r}_m$ . To support our proposal for the use of the  $\mathbf{w}_{\text{MVDR}}$  filter in the DPSK/CDMA demodulator of Fig. 1, we need the following proposition.

**PROPOSITION 1.**

(i) *If the information bits  $\{b_k(m)\}$ ,  $m = 1, 2, \dots$ , of the  $k$ -th user,  $k = 0, 1, \dots, K - 1$ , are independent and uniformly distributed, then the transmitted differential bits  $\{d_k(m)\}$ ,  $m = 1, 2, \dots$ , are also independent and uniformly distributed (despite the recursive generation procedure  $d_k(m) = b_k(m)d_k(m - 1)$ ).*

(ii) *In the absence of Inter-Symbol-Interference (ISI), the variance at the output of the differential logic is equal to the square of the variance at the input of the differential logic, i.e.*

$$E \left\{ |y_m|^2 \right\} = \left( E \left\{ |x_m|^2 \right\} \right)^2. \quad \square \quad (22)$$

The proof of (i) is based on recursion (3) and the statistical independence between  $b_k(m)$  and  $d_k(m - 1)$ ,  $k = 0, 1, \dots, K - 1$ ,  $m = 1, 2, \dots$ . For the proof of (ii), we condition the output variance  $E \left\{ |y_m|^2 \right\}$  on  $\mathbf{d}(m)$  and  $\mathbf{d}(m - 1)$ . Given this condition, the successive differential logic inputs  $x_m$  and  $x_{m-1}$  become statistically independent and the proof is completed with the help of (i).

In the absence of ISI among two successive received data vectors  $\mathbf{r}_{m-1}$  and  $\mathbf{r}_m$ , Part (ii) of Proposition 1 shows that  $\mathbf{w}_{\text{MVDR}}$  in (21), originally designed to minimize the filter-output (differential-logic-input) disturbance variance, also minimizes the disturbance variance at the output of the differential logic of the structure in Fig. 1. However, under the multipath fading model of Section 2 the  $\mathbf{w}_{\text{MVDR}}$  filter is no longer the optimum linear filter in the minimum-differential-logic-output-variance sense.

To compute the minimum-differential-logic-output-variance filter  $\mathbf{w}_{\text{MDLOV}}$  we have to minimize the variance expression  $E \left\{ |y_m|^2 \right\} = E \left\{ \mathbf{w}^H \mathbf{r}_m \mathbf{r}_m^H \mathbf{w} \mathbf{w}^H \mathbf{r}_{m-1} \mathbf{r}_{m-1}^H \mathbf{w} \right\}$  with respect to  $\mathbf{w}$ , subject to the signal preservation constraint  $\mathbf{w}^H \mathbf{V}_0 = 1$ . Unfortunately, constrained multi-modal minimization problems of this type do not have in general closed-form analytic solutions and constrained stochastic-approximation-type recursive solutions are not guaranteed to converge to the global minimum  $\mathbf{w}_{\text{MDLOV}}$ . Alternatively, we may consider a subclass of time-domain ISI excision filters  $\mathbf{w} \in \mathcal{C}^{L+N-1}$  by setting  $\mathbf{w}[0] = \mathbf{w}[1] = \dots = \mathbf{w}[N-2] = 0$  and  $\mathbf{w}[L] = \mathbf{w}[L+1] = \dots = \mathbf{w}[L+N-2] = 0$ . The filters in this class achieve ISI excision at the cost of rejecting information bearing data (only  $L-N+1$  samples from the  $(L+N-1)$ -dimensional vector  $\mathbf{r}_m$  are utilized in the demodulation/detection process). The unique optimum filter in this class  $\mathbf{w}_{\text{MDLOV}}^{\text{excise}} = \mathbf{w}_{\text{MVDR}}^{\text{excise}}$  can be easily derived analytically. Nevertheless, our simulation results reveal that, for all practical purposes,  $\mathbf{w}_{\text{MDLOV}}$  and  $\mathbf{w}_{\text{MVDR}}$  exhibit very similar Bit-Error-Rate performance and are superior to the excision filter  $\mathbf{w}_{\text{MDLOV}}^{\text{excise}}$ . Taking into account these findings, our analysis will focus on the estimation of the minimum-filter-output-variance  $\mathbf{w}_{\text{MVDR}}$  filter.

Adaptive implementations of the  $\mathbf{w}_{\text{MVDR}}$ /DPSK/CDMA demodulator require a (possibly phase ambiguous) RAKE estimate  $\hat{\mathbf{V}}_0$  and an estimate of the input autocorrelation matrix  $R = E \left\{ \mathbf{r}_m \mathbf{r}_m^H \right\}$ . When the latter is obtained through sample averaging over a data record of  $J$  inputs,

$$\hat{R}(J) = \frac{1}{J} \sum_{j=1}^J \mathbf{r}_j \mathbf{r}_j^H, \quad (23)$$

$\hat{\mathbf{w}}_{\text{MVDR}}(J) \triangleq \frac{\hat{R}(J)^{-1} \hat{\mathbf{V}}_0}{\hat{\mathbf{V}}_0^H \hat{R}(J)^{-1} \hat{\mathbf{V}}_0}$  is known as the Sample-Matrix-Inversion (SMI) MVDR filter estimate.

The following section is devoted entirely to distortionless filters and minimum variance optimization. However, in that section we are willing to trade “*filter-estimator bias*” (in terms of statistical minimum variance optimality) for lower computational optimization complexity (no matrix inversions as in (21)) and lower short-data-record “*filter-estimator variance*”. The latter is of utmost importance when short-data-record adaptive DPSK/CDMA demodulation is pursued in an effort to catch-up with time-varying communication channels.

## 5. SHORT-DATA-RECORD ADAPTIVE DPSK/CDMA DEMODULATORS

Without loss of generality we assume that the phase ambiguous RAKE vector  $\mathbf{V}_0$  for the user of interest 0 is normalized. Starting from

$$\mathbf{w}_0 \triangleq \mathbf{V}_0, \quad (24)$$

we intend to generate an *infinite* sequence  $\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \dots$  of linear pre-filter candidates for the demodulator in Fig. 1 that are all “distortionless” in the  $\mathbf{V}_0$  vector direction ( $\mathbf{w}_p^H \mathbf{V}_0 = 1, p = 0, 1, \dots$ ). The filter sequence is created by the simple recursion

$$\mathbf{w}_p = \mathbf{w}_{p-1} - \mu_p \mathbf{G}_p, \quad p = 1, 2, \dots, \quad \mathbf{w}_0 = \mathbf{V}_0 \quad (25)$$

where: (i)  $\mathbf{G}_p, p = 1, 2, \dots$ , are  $(L+N-1)$ -dimensional orthonormal to  $\mathbf{V}_0$  vectors ( $\mathbf{G}_p^H \mathbf{V}_0 = 0, \mathbf{G}_p^H \mathbf{G}_p = 1$ ) to be defined and (ii)  $\mu_p, p = 1, 2, \dots$ , are scalars to be defined. Our choice for  $\mathbf{G}_p, \mu_p, p = 1, 2, \dots$ , relies strictly on *conditional* statistical optimization principles that lead to the following inductive definition. Assume that  $\mathbf{w}_{p-1}$  is defined for some  $p \geq 1$  and  $\mathbf{w}_{p-1} \neq \mathbf{w}_{\text{MVDR}}$ . We view  $\mathbf{G}_p$  as an “auxiliary vector” that assists  $\mathbf{w}_{p-1}$  in further suppressing input interference. The auxiliary vector design criterion<sup>5,12</sup> is the maximization of the magnitude of the cross-correlation between the  $\mathbf{G}_p$  processed and the  $\mathbf{w}_{p-1}$  processed input data vectors  $\mathbf{r}_m$ , subject to the orthonormality constraint of  $\mathbf{G}_p$  with respect to  $\mathbf{V}_0$ :

$$\mathbf{G}_p = \arg \max_{\mathbf{G}} \left| E \left\{ (\mathbf{w}_{p-1}^H \mathbf{r}_m) (\mathbf{G}^H \mathbf{r}_m)^* \right\} \right| = \arg \max_{\mathbf{G}} \left| \mathbf{w}_{p-1}^H R \mathbf{G} \right|, \quad (26)$$

subject to  $\mathbf{G}^H \mathbf{G} = 1, \quad \mathbf{G}^H \mathbf{V}_0 = 0.$

Particularly, without loss of generality, we seek the unique vector  $\mathbf{G}_p$  which is a solution to our constraint optimization problem and makes  $\mathbf{w}_{p-1}^H R \mathbf{G}$  real non-negative ( $\mathbf{w}_{p-1}^H R \mathbf{G} \geq 0$ ). This auxiliary vector is<sup>5,12,13</sup>

$$\mathbf{G}_p = \frac{R \mathbf{w}_{p-1} - (\mathbf{V}_0^H R \mathbf{w}_{p-1}) \mathbf{V}_0}{\|R \mathbf{w}_{p-1} - (\mathbf{V}_0^H R \mathbf{w}_{p-1}) \mathbf{V}_0\|}. \quad (27)$$

Given  $\mathbf{w}_{p-1}$  and  $\mathbf{G}_p$ , the complex scalar  $\mu_p$  is naturally chosen to be the value that minimizes the output variance of the  $\mathbf{w}_p = \mathbf{w}_{p-1} - \mu_p \mathbf{G}_p$  filter. The minimum-variance optimum value  $\mu_p$  is found to be<sup>5,10-13</sup>

$$\mu_p = \frac{\mathbf{G}_p^H R \mathbf{w}_{p-1}}{\mathbf{G}_p^H R \mathbf{G}_p}. \quad (28)$$

We note that the computation of the filter  $\mathbf{w}_p$  through recursion (25) and (27), (28) requires no matrix inversion operation. By inspection, we also observe that for the MS-optimum value of  $\mu_p$ , the product  $\mu_p \mathbf{G}_p = \frac{\mathbf{G}_p^H R \mathbf{w}_{p-1}}{\mathbf{G}_p^H R \mathbf{G}_p} \mathbf{G}_p$  is independent of the norm of  $\mathbf{G}_p$ . Hence, so is  $\mathbf{w}_p$ . Therefore, we may drop the unnecessary normalization of the auxiliary vectors. We can also factorize the auxiliary vector numerator to make the  $\mathbf{V}_0$  orthogonal projection operator apparent. Then, the expression for the  $p$ -th auxiliary vector becomes

$$\mathbf{G}_p = (I - \mathbf{V}_0 \mathbf{V}_0^H) R \mathbf{w}_{p-1}, \quad p = 1, 2, \dots \quad (29)$$

It is important to note that, while the generated auxiliary vectors  $\mathbf{G}_1, \mathbf{G}_2, \dots$  are all constrained to be orthogonal to  $\mathbf{V}_0$ , orthogonality among the auxiliary vectors is *not* imposed. This is in sharp contrast to previous work that involved filtering with up to  $L + N - 2$  orthogonal to each other and to  $\mathbf{V}_0$  auxiliary vectors (AVs).<sup>12</sup> Formal theoretical analysis of the sequence of auxiliary-vector filters  $\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \dots$  was pursued in Ref. 13 and led to the results summarized below in the form of a proposition.

**PROPOSITION 2.** *Let  $R$  be a Hermitian positive definite matrix. Consider the auxiliary-vector filter sequence  $\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \dots$  defined by (25), (29), and (28).*

- (i) *Successive auxiliary vectors generated through (25), (29), and (28) are orthogonal:  $\mathbf{G}_p^H \mathbf{G}_{p+1} = 0, p = 1, 2, 3, \dots$*
- (ii) *The generated sequence of auxiliary-vector weights  $\{\mu_p\}, p = 1, 2, \dots$ , is real-valued, positive, and bounded:*

$$0 < \frac{1}{\lambda_{\max}} \leq \mu_p \leq \frac{1}{\lambda_{\min}}, \quad p = 1, 2, \dots, \quad (30)$$

where  $\lambda_{\max}$  and  $\lambda_{\min}$  are the maximum and minimum, correspondingly, eigenvalues of  $R$ .

- (iii) *The sequence of auxiliary vectors  $\{\mathbf{G}_p\}, p = 1, 2, \dots$ , converges to the  $\mathbf{0}$  vector:*

$$\lim_{p \rightarrow \infty} \mathbf{G}_p = \mathbf{0}. \quad (31)$$

- (iv) *The sequence of auxiliary-vector filters  $\mathbf{w}_0, \mathbf{w}_1, \mathbf{w}_2, \dots$  converges to the MVDR filter:*

$$\lim_{p \rightarrow \infty} \mathbf{w}_p = \mathbf{w}_{\text{MVDR}} = \frac{R^{-1} \mathbf{V}_0}{\mathbf{V}_0^H R^{-1} \mathbf{V}_0}. \quad \square \quad (32)$$

If the input autocorrelation matrix  $R$  that appears in (29) and (28) is perfectly known, then, according to Proposition 2, Part (iv), the sequence  $\{\mathbf{w}_p\}$  converges to the MVDR filter  $\mathbf{w}_{\text{MVDR}}$ . Certainly, in our DPSK/CDMA problem we assume that  $R$  is unknown and it is sample-average estimated by  $\hat{R}(J)$  in (23). In that case, the sequence of auxiliary-vector filter *estimators*  $\{\hat{\mathbf{w}}_p(J)\}$  converges to the SMI filter estimate  $\hat{\mathbf{w}}_{\text{MVDR}}(J) = \frac{\hat{R}(J)^{-1} \mathbf{V}_0}{\mathbf{V}_0^H \hat{R}(J)^{-1} \mathbf{V}_0}$ :  $\lim_{p \rightarrow \infty} \hat{\mathbf{w}}_p(J) = \hat{\mathbf{w}}_{\infty}(J) = \hat{\mathbf{w}}_{\text{MVDR}}(J)$ .

It is much more productive, however, to switch our attention from the limit filter estimator  $\hat{\mathbf{w}}_{\infty}(J)$  to the other regular members of the auxiliary-vector (AV) filter estimator sequence. The sequence begins from  $\hat{\mathbf{w}}_0(J) = \mathbf{V}_0$ , which is a  $\theta$ -variance, fixed-valued, estimator that may be severely biased ( $\hat{\mathbf{w}}_0(J) = \mathbf{V}_0 \neq \mathbf{w}_{\text{MVDR}}$ ) unless  $R = \sigma^2 I$ , for some

$\sigma > 0$ . In the latter trivial case,  $\hat{\mathbf{w}}_0(J)$  is already the perfect MVDR filter. Otherwise, the next filter estimator in the sequence,  $\hat{\mathbf{w}}_1(J)$ , has a significantly reduced bias due to the greedy optimization procedure employed, at the expense of non-zero estimator (co-)variance. As we move up in the sequence of filter estimators  $\hat{\mathbf{w}}_p(J)$ ,  $p = 0, 1, 2, \dots$ , the bias decreases rapidly to zero while the variance raises gracefully to the SMI ( $\hat{\mathbf{w}}_\infty(J)$ ) levels.<sup>13</sup> To choose “the best” auxiliary-vector filter estimator in the minimum Mean-Square estimation error sense, we have to balance the low-variance, low-bias requirements for a given data record size  $J$ . In that sense, for short-data-record rapid adaptive system optimization, AV filter estimators that are among the first few members of the filter sequence  $\{\hat{\mathbf{w}}_p(J)\}$  offer superior system BER performance as seen in the next section.

## 6. NUMERICAL AND SIMULATION STUDIES

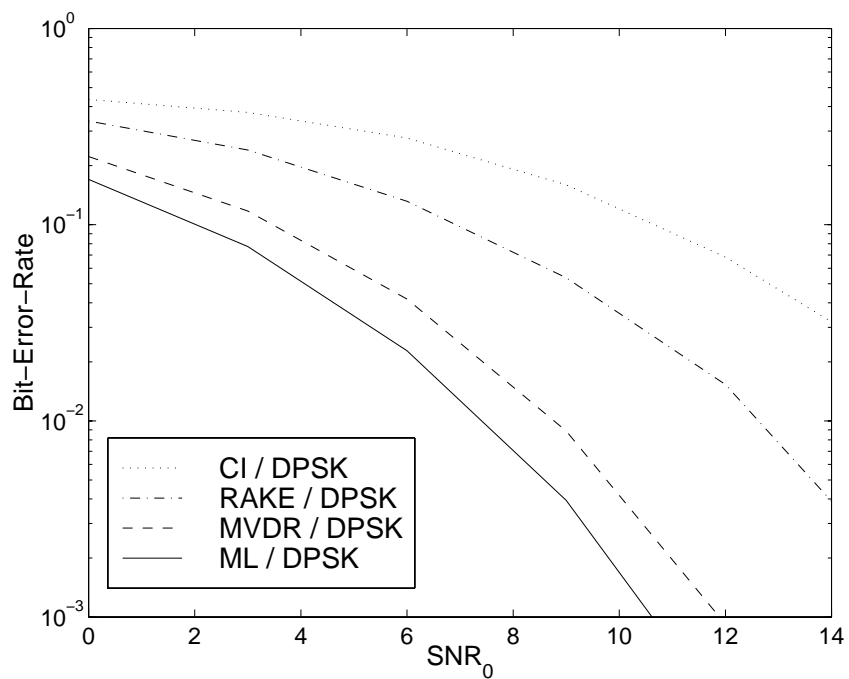
We consider the multipath DPSK DS/CDMA signal model of Section 2 with spreading gain  $L = 31$ . We assume the presence of  $K = 3$  active users with fixed signature assignments and normalized synchronous signature cross-correlations given by  $\mathbf{S}^T \mathbf{S} = \begin{bmatrix} 1 & 0.22 & 0.29 \\ 0.22 & 1 & 0.09 \\ 0.29 & 0.09 & 1 \end{bmatrix}$ , where  $\mathbf{S}$  is the  $L \times 3$  matrix with columns the normalized signature vectors  $\mathbf{S}_0$ ,  $\mathbf{S}_1$ , and  $\mathbf{S}_2$ . Each user signal experiences  $N = 4$  independent paths and the corresponding channel coefficients are assumed to be zero-mean complex Gaussian random variables of equal variance. Then, following the notation of Section 2, the *total received SNR* for User  $k$ ,  $k = 0, 1, 2$ , is  $\text{SNR}_k \triangleq \frac{E_k \sum_{n=0}^3 E\{|\alpha_{k,n}|^2\}}{N_0}$  (Ref. 1).

In Fig. 2 we fix the total SNR of the two interferers at  $\text{SNR}_1 = 10\text{dB}$  and  $\text{SNR}_2 = 11\text{dB}$ , respectively, and we compare the BER versus  $\text{SNR}_0$  performance of the following receiver structures: (a) The RAKE/DPSK receiver that we defined through Fig. 1 and (20), (b) the ideal (perfectly known  $R$ ) MVDR/DPSK receiver given by Fig. 1 and (21), (c) the ML/DPSK detector derived in (18) of Section 3, and (d) the channel-independent multipath DPSK demodulator as defined in Ref. 1, that we call CI/DPSK. We recall that the latter CI/DPSK receiver does not require/utilize a channel estimate at all, while the RAKE and the MVDR DPSK receivers require a phase ambiguous estimate of  $\mathbf{V}_0$ . The ML detector assumes availability of the channel coefficients of *all* user signals within a common phase ambiguity (second-order combined multi-channel estimation). The results that we present are uncoded (“raw”) BERs for DPSK transmissions over multipath fading channels and are averages over 100 independent Rayleigh channel realizations. As expected, the ML/DPSK receiver establishes the BER lower bound for all other structures. However, the fact that the MVDR/DPSK demodulator is within 2dB or less from the ML detector appears particularly encouraging.

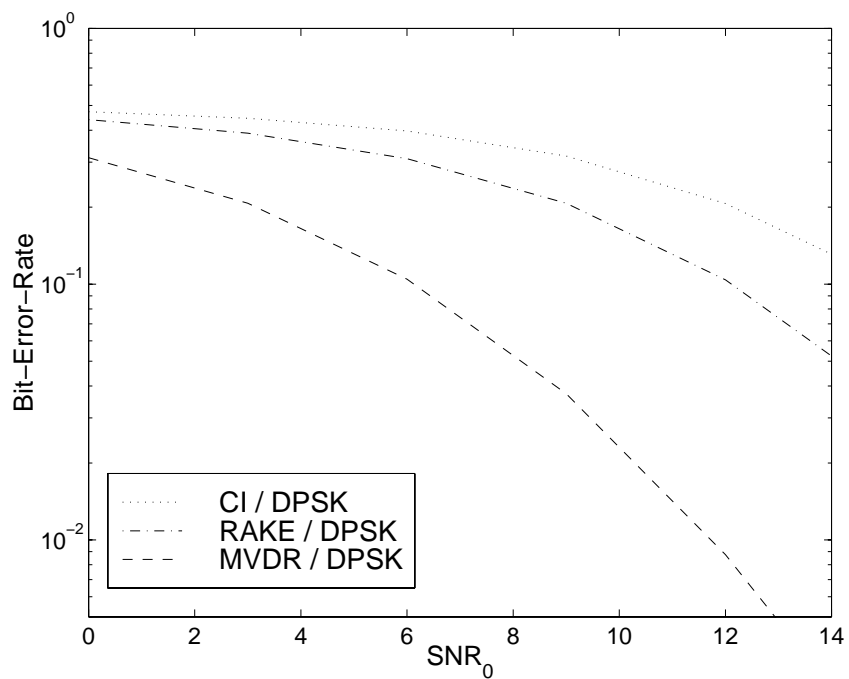
In Fig. 3 we extend our study to a more realistic scenario and we restrict our attention to linear pre-filter DPSK demodulation only. Gold sequences of length  $L = 31$  are assigned to  $K = 13$  active users. The total received SNR of the interferers  $k = 1, 2, \dots, 12$  is set at  $\text{SNR}_{1-4} = 8\text{dB}$ ,  $\text{SNR}_{5-8} = 10\text{dB}$ , and  $\text{SNR}_{9-12} = 12\text{dB}$ . The BER results that we present are again averages over 100 independent Rayleigh channel realizations. We notice that the MVDR/DPSK demodulator maintains similar performance to the case study of Fig. 2. However, this “interference-loaded” set-up affects significantly the performance of the CI/DPSK and RAKE/DPSK receivers. For these studies we assumed perfect knowledge of the received data vector autocorrelation matrix  $R$  that is needed for the design of the MVDR filter in (21).

In Fig. 4 we maintain the signal model of Fig. 3 and we consider for the first time adaptive sample average estimates of  $R$  as in (23) with  $J = 150$  data vectors. The induced BERs are averages over 100 independently drawn Rayleigh channels and, when relevant, 5 independent adaptive filter realizations per channel. We observe that the SMI filter estimator suffers from severe “data starvation”. This is not the case for the AV filter estimators that are seen to offer substantial performance improvement in comparison with the RAKE and SMI pre-filters.

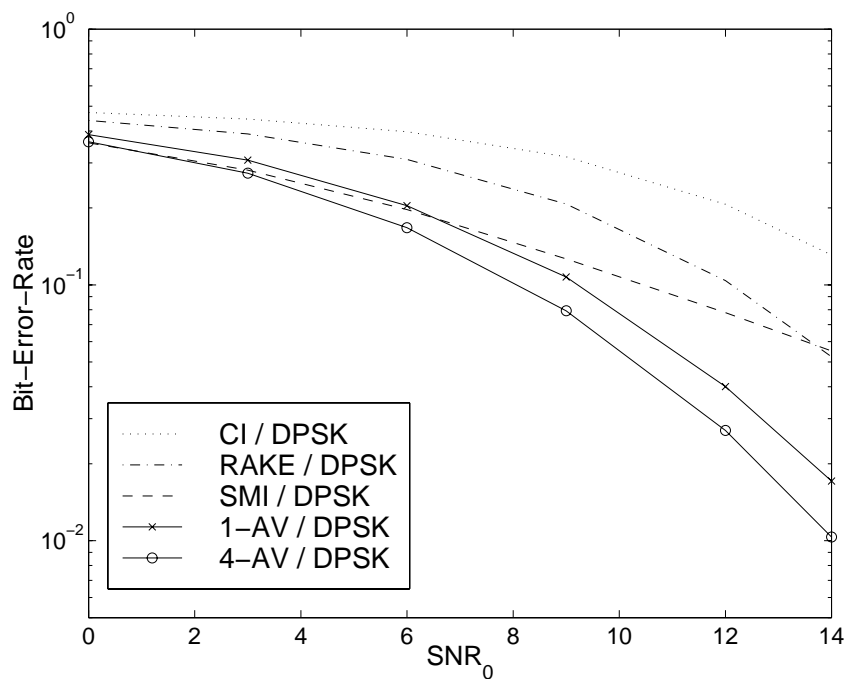
Finally, in Fig. 5 we fix the SNR of the user of interest at  $\text{SNR}_0 = 12\text{dB}$  and we examine the BER of the SMI, 1-AV, 4-AV, and 6-AV DPSK demodulators as a function of the sample support. The non-adaptive CI and RAKE/DPSK demodulators are included as a reference. The superiority of the 4-AV/DPSK receiver is apparent over the 50-to-200 data-support range of practical interest.



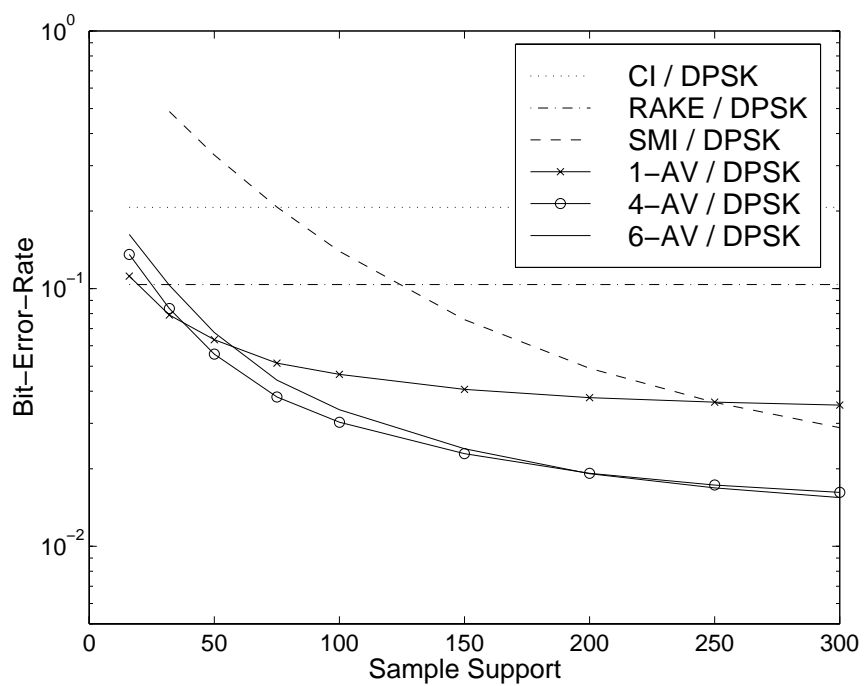
**Figure 2.** BER versus total SNR for the CI/DPSK, RAKE/DPSK, MVDR/DPSK, and ML/DPSK demodulators ( $K = 3$  users).



**Figure 3.** BER versus total SNR for the CI/DPSK, RAKE/DPSK, and MVDR/DPSK demodulators ( $K = 13$  users with Gold signatures).



**Figure 4.** BER versus total SNR for the CI/DPSK, RAKE/DPSK and the SMI/DPSK, AV/DPSK (with one and four auxiliary vectors) adaptive receivers. The input autocorrelation matrix is estimated from  $J = 150$  samples and the system set-up is as in Fig. 3.



**Figure 5.** BER versus sample support studies (system set-up as in Figs. 3 and 4).

## 7. CONCLUSIONS

The basic, core application that led us to this present study was PSK communications over slowly flat-fading channels. No resource is to be overlooked in fighting the severe effect of fading on PSK communications systems. In the presence of independent multipath slow flat-fading, multipath combining is an opportunity not to be wasted. However, successful adaptive multipath channel estimation algorithms are based on second-order statistics and return phase ambiguous channel impulse response estimates. This is perfectly tolerable under Differential PSK (DPSK) modulation.

To account for multiple-access communications, we considered a DPSK DS/CDMA communications system. We derived the multiuser single-symbol (1-lag) Maximum Likelihood DPSK/CDMA demodulator with direct generalization to multiple-symbol (block) differential demodulation. As expected, the computational complexity of the ML decision rule is exponential in the number of CDMA users and in the block length. This limits the importance of this development to theoretical lower-bound Bit-Error-Rate benchmarking. In search of practical solutions, we withdrew the ML design criterion and focused on DPSK/CDMA demodulators with Multiple-Access-Interference (MAI) suppression in the form of linear pre-filtering. RAKE (phase ambiguous) filtering and RAKE output differential detection is MS-optimum only for antipodal single-user transmissions or under perfect orthogonality with respect to all other user RAKE filters. The adaptive minimum output variance filter (MVDR) that is distortionless in the phase ambiguous RAKE vector direction of interest was shown to minimize the disturbance variance at the *differential logic output* in the absence of Inter-Symbol-Interference. Even when ISI is present, the MVDR filter and the hard-to-obtain minimum-differential-logic-output-variance linear filter exhibit very similar performance characteristics. Preferable alternatives of very low optimization complexity (no matrix inversion operations) and superior BER performance under rapid short-data-record adaptation were identified in the class of Auxiliary-Vector (AV) filters.

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