New Bounds on the Total Squared Correlation and Optimum Design of DS-CDMA Binary Signature Sets

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***Abstract*—The Welch lower bound on the total squared correlation (TSC) of signature sets is known to be tight for real-valued signatures and loose for binary signatures whose number is not a multiple of four. In this letter, we derive new bounds on the TSC of binary signature sets for any number of signatures K and any signature length L. Then, for almost all K, L in $\{1, 2, \ldots, 256\}$, we design optimum binary signature sets that achieve the new bounds. The design procedure is based on simple transformations of Hadamard matrices.**

***Index Terms*—Binary sequences, code-division multiple access (CDMA), codes, signal design.**

I. New Bounds on the TSC of Binary Antipodal Signature Sets

IN DIRECT-SEQUENCE code-division-multiple-access (DS-CDMA) systems, multiple users are assigned individual binary antipodal signatures (spreading codes) to access a common (in time and frequency) communication channel. A fundamental measure of the cross-correlation properties of a signature set is the total squared correlation (TSC).[1] If $\mathcal{S} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K\}$, $\mathbf{s}_i \in \mathbb{C}^L$, $\|\mathbf{s}_i\| = 1$, $i = 1, 2, \ldots, K$, is a set of K normalized (complex valued in general) user signatures of length (processing gain) L, then the TSC of set $\mathcal{S}$ is the sum of the squared magnitudes of all inner products between signatures [2]

$$\text{TSC}(\mathcal{S}) \stackrel{\triangle}{=} \sum_{i=1}^{K} \sum_{j=1}^{K} |\mathbf{s}_i^H \mathbf{s}_j|^2. \tag{1}$$

Welch showed [3] that $\text{TSC}(\mathcal{S}) \geq K^2/L$, and this lower bound was named [2] the "Welch bound" on the TSC of signature sets. We know that if $K \geq L$, then there always exists [4] a *real-valued* signature set that yields equality in the Welch bound.[2] Such optimum sets are called Welch-bound-equality (WBE) signature sets [2]. Algorithms for the generation of real-valued WBE signature sets are developed in [5]–[7].

While for real/complex-valued signature sets, the Welch bound (K^2/L for $K \geq L$ and K for $K < L$) is always achievable, this is not the case for binary antipodal signature sets. In [2], it is stated that if $K \geq L$, the Welch bound K^2/L is achieved with binary antipodal signatures only if $K \equiv 0$ (mod 4) or K =1 or 2. For the case of $K < L$, the lower bound K is achieved only if $L \equiv 0$ (mod 4) or L =1 or 2. Optimum binary antipodal signature sets are constructed in [2], [8], and [9] when the number of users is a power of two and equals or exceeds the system processing gain ($K = 2^n \geq L, n \leq L$).

In this letter, first we derive new bounds on the TSC of binary antipodal signature sets for all possible combinations of the values of K (number of users) and L (processing gain). Then, for most K, L, we design via simple Hadamard matrix transformations optimum binary signature sets that achieve the new bounds. The K, L optimum design cases that are not covered under these developments—therefore, the tightness of the corresponding bounds is not established—are $L = K \equiv 1 \, (\text{mod}\, 4)$; $L = K \equiv 2 \, (\text{mod}\, 4)$; $L + 1 = K \equiv 2 \, (\text{mod}\, 4)$; and $K + 1 = L \equiv 2 \, (\text{mod}\, 4)$.

We consider a binary antipodal signature set $\mathcal{S} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K\}$ with K normalized signatures $\mathbf{s}_i \in \{\pm 1/\sqrt{L}\}^L$, $i = 1, 2, \ldots, K$, and we begin our presentation with the derivation of the new bounds for the "underloaded" $(K \leq L)$ system case. The "overloaded" $(K \geq L)$ system case follows.

A. Underloaded System $(K \leq L)$

The TSC of $\mathcal{S}$ is $\text{TSC}(\mathcal{S}) = \sum_{i=1}^{K} \sum_{j=1}^{K} (\mathbf{s}_i^T \mathbf{s}_j)^2 = K + \sum_{i=1}^{K} \sum_{j=1, j \neq i}^{K} (\mathbf{s}_i^T \mathbf{s}_j)^2$. The second, double-summation term is the TSC between different users in $\mathcal{S}$. To obtain a bound on this term, we state and prove the following theorem.

Theorem 1 (On the Cross-Correlation Characteristics of Binary Antipodal Signature Sets): Let $\mathcal{S} = \{\mathbf{s}_i\}_{i=1}^{K}$ be a binary antipodal signature set where $\mathbf{s}_i \in \{\pm A\}^L$, $i = 1, 2, \ldots, K$, $K \leq L$, and $A \in \mathbb{C} - \{0\}$.

1) If $\mathbf{s}_i^H \mathbf{s}_j \neq 0$, then

$$|\mathbf{s}_i^H \mathbf{s}_j| \geq \begin{cases} 2|A|^2, & L \equiv 0 \, (\text{mod}\, 2) \\ |A|^2, & L \equiv 1 \, (\text{mod}\, 2), \end{cases} \qquad 1 \leq i, j \leq K. \tag{2}$$

2) Consider the set C of all nonordered pairs of signatures $\{\mathbf{s}_i, \mathbf{s}_j\}$, $i \neq j$, with nonzero cross correlation: $C(\{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_K\}) \stackrel{\triangle}{=} \{\{\mathbf{s}_i, \mathbf{s}_j\}$ such that $i \neq j$ and

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[1]The term total squared correlation and the acronym TSC are due to [1].

[2]If $K < L$, the Welch bound K^2/L becomes loose and a tighter bound exists: $\text{TSC}(\mathcal{S}) \geq K$. In that case, the bound value K can be trivially achieved by any orthonormal set of K real/complex-valued signatures of length L.

$\mathbf{s}_i^H\mathbf{s}_j \neq 0, i = 1,2,\ldots,K, j = 1,2,\ldots,K\}$. If $|C|$ denotes the cardinality of set C, then

$$|C(\{\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_K\})| \geq \begin{cases} 0, & L\equiv 0 (\mathrm{mod}\ 4)\\ \frac{K(K-2)}{4}, & L\equiv 2 (\mathrm{mod}\ 4) \\ & \text{and} K\equiv 0 (\mathrm{mod}\ 2) \\ \frac{(K-1)^2}{4}, & L\equiv 0 (\mathrm{mod}\ 4) \\ & \text{and} K\equiv 1 (\mathrm{mod}\ 2) \\ \frac{K(K-1)}{2}, & L\equiv 1 (\mathrm{mod}\ 2). \end{cases} \quad (3)$$

Proof: With respect to part 1), if $L \equiv 0 \pmod 2$, then $L = 2m, m \in \{1,2,\ldots\}$, and $|\mathbf{s}_i^H\mathbf{s}_j| = |L-2d(\mathbf{s}_i,\mathbf{s}_j)|\,|A|^2 = 2|m-d(\mathbf{s}_i,\mathbf{s}_j)|\,|A|^2$ where $d(\mathbf{s}_i,\mathbf{s}_j)$ denotes the Hamming distance between $\mathbf{s}_i,\mathbf{s}_j \in \{\pm A\}^L$. Therefore, if $\mathbf{s}_i^H\mathbf{s}_j \neq 0$ and $L \equiv 0 \pmod 2$, then $|\mathbf{s}_i^H\mathbf{s}_j| \geq 2|A|^2$. If, on the other hand, $L \equiv 1 \pmod 2$, then $L = 2m+1$, $m \in \{0,1,2,\ldots\}$, and $|\mathbf{s}_i^H\mathbf{s}_j| = |L-2d(\mathbf{s}_i,\mathbf{s}_j)|\,|A|^2 = |2(m-d(\mathbf{s}_i,\mathbf{s}_j))+1|\,|A|^2$. Therefore, if $L \equiv 1 \pmod 2$, then $|\mathbf{s}_i^H\mathbf{s}_j| \geq |A|^2$ for all $i \neq j$.

With respect to part 2), if $L \equiv 0 \pmod 4$, then $|C(\mathcal{S})| \geq 0$ trivially by definition. If $L \equiv 1 \pmod 2$, then $\mathbf{s}_i^H\mathbf{s}_j \neq 0$ for all $i \neq j$ [as shown in the proof of 1)]. Therefore, $|C(\mathcal{S})| = \binom{K}{2} = K(K-1)/2$. If $L \equiv 2 \pmod 4$, then we choose an arbitrary signature from set $\mathcal{S}$, say $\mathbf{s}_1$, and we partition $\mathcal{S}$ into two disjoint sets as follows: $\mathcal{S}_1 \triangleq \{\mathbf{s}_i : d(\mathbf{s}_i,\mathbf{s}_1) \equiv 0 \pmod 2, i = 1,2,\ldots,K\}$ and $\mathcal{S}_2 \triangleq \{\mathbf{s}_i : d(\mathbf{s}_i,\mathbf{s}_1) \equiv 1 \pmod 2, i = 1,2,\ldots,K\}$. We can show that there is no pair of signatures with zero cross correlation within $\mathcal{S}_1$ or within $\mathcal{S}_2$.[3] Let $|\mathcal{S}_1| = k$ and $|\mathcal{S}_2| = K-k$, $0 \leq k \leq K$. Then, $|C(\mathcal{S}_1)| = \binom{k}{2}$ and $|C(\mathcal{S}_2)| = \binom{K-k}{2}$. Since $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$ and the two subsets $\mathcal{S}_1$ and $\mathcal{S}_2$ are disjoint, we obtain $|C(\mathcal{S})| \geq \binom{k}{2} + \binom{K-k}{2} = k^2 - Kk + K(K-1)/2$. If $K \equiv 0 \pmod 2$, then the right-hand side of this inequality is minimized for $k = K/2 : |C(\mathcal{S})| \geq (K/2)^2 - K(K/2) + K(K-1)/2 = K(K-2)/4$. If $K \equiv 1 \pmod 2$, then the right-hand side is minimized for $k = (K \pm 1)/2$: $|C(\mathcal{S})| \geq ((K\pm 1)/2)^2 - K(K\pm 1)/2 + K(K-1)/2 = (K-1)^2/4$.□

If the processing gain L is even, by direct application of *Theorem 1*, part 1), we obtain $\mathrm{TSC}(\mathcal{S}) \geq K + 2|C(\{\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_K\})|(2|1/\sqrt{L}|^2)^2 = K + (8/L^2)|C(\{\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_K\})|$. Then, *Theorem 1*, part 2), gives

$$\mathrm{TSC}(\mathcal{S}) \geq \begin{cases} K, & L\equiv 0 \pmod 4 \\ K + 2\frac{K(K-2)}{L^2}, & L\equiv 2 \pmod 4 \\ & \text{and } K\equiv 0 \pmod 2 \\ K + 2\left(\frac{K-1}{L}\right)^2, & L\equiv 2 \pmod 4 \\ & \text{and } K\equiv 1 \pmod 2. \end{cases} \quad (4)$$

If L is odd, then from *Theorem 1*, part 1), $\mathrm{TSC}(\mathcal{S}) \geq K + 2|C(\{\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_K\})|(|1/\sqrt{L}|^2)^2 = K + (2/L^2)|C(\{\mathbf{s}_1,\mathbf{s}_2,\ldots,\mathbf{s}_K\})|$ and from *Theorem 1*, part 2), we obtain

$$\mathrm{TSC}(\mathcal{S}) \geq K + \frac{K(K-1)}{L^2}, \qquad L \equiv 1 \pmod 2. \quad (5)$$

Equations (4) and (5) define the new bounds on the TSC of binary antipodal signature sets for underloaded $(K \leq L)$ systems and are summarized in Table I. Table I can also be seen as a proof that when the number of users is more than two and the signature length is not a multiple of four, no orthogonal binary antipodal signature set exists. In addition, it is interesting to note that the familiar Gold signature sets [10] with $K \leq L$, $L = 2^p - 1$, $p = 5,6,\ldots$, as well as the signature sets obtained by $K \leq L$ cyclic shifts of an m-sequence [11] of length $L = 2^p - 1$, $p = 3,4,\ldots$, meet the bound in (5) with equality; hence, both sets are minimum-TSC optimum.

[3]If $\mathbf{s}_i,\mathbf{s}_j \in \mathcal{S}_1$ (or $\mathcal{S}_2$), we can show that $d(\mathbf{s}_i,\mathbf{s}_j) \equiv 0 \pmod 2$. Since $L \equiv 2 \pmod 4$ and $d(\mathbf{s}_i,\mathbf{s}_j) \equiv 0 \pmod 2$, we can show that $\mathbf{s}_i^H\mathbf{s}_j \neq 0$ for all $\mathbf{s}_i,\mathbf{s}_j \in \mathcal{S}_1$ (or $\mathcal{S}_2$).

TABLE I
UNDERLOADED DS-CDMA SYSTEM $(K \leq L)$

Processing Gain	Number of Users	Lower Bound on TSC
$L \equiv 0 \pmod 4$	Any K	K
$L \equiv 2 \pmod 4$	$K \equiv 0 \pmod 2$	$K + 2\frac{K(K-2)}{L^2}$
	$K \equiv 1 \pmod 2$	$K + 2\left(\frac{K-1}{L}\right)^2$
$L \equiv 1 \pmod 2$	Any K	$K + \frac{K(K-1)}{L^2}$

TABLE II
OVERLOADED DS-CDMA SYSTEM $(K \geq L)$

Number of Users	Processing Gain	Lower Bound on TSC
$K \equiv 0 \pmod 4$	Any L	$\frac{K^2}{L}$
$K \equiv 2 \pmod 4$	$L \equiv 0 \pmod 2$	$\frac{K^2}{L} + 2\frac{L-2}{L}$
	$L \equiv 1 \pmod 2$	$\frac{K^2}{L} + 2\left(\frac{L-1}{L}\right)^2$
$K \equiv 1 \pmod 2$	Any L	$\frac{K^2}{L} + \frac{L-1}{L}$

B. *Overloaded System* $(K \geq L)$

Let $\mathbf{d}_l \triangleq [\mathbf{s}_1(l),\mathbf{s}_2(l),\ldots,\mathbf{s}_K(l)]^T \in \{\pm 1/\sqrt{L}\}^K$ denote the transpose of the lth row, $l = 1,2,\ldots,L$, of the signature matrix. Then, due to the "row-column equivalence" [2], $\mathrm{TSC}(\mathcal{S}) = \sum_{i=1}^K\sum_{j=1}^K(\mathbf{s}_i^T\mathbf{s}_j)^2 = \sum_{l=1}^L\sum_{m=1}^L(\mathbf{d}_l^T\mathbf{d}_m)^2$. Therefore, we can proceed with the calculation of the TSC of $\mathcal{S}$ as follows: $\mathrm{TSC}(\mathcal{S}) = \sum_{l=1}^L(\mathbf{d}_l^T\mathbf{d}_l)^2 + \sum_{l=1}^L\sum_{m=1,m\neq l}^L(\mathbf{d}_l^T\mathbf{d}_m)^2 = K^2/L + \sum_{l=1}^L\sum_{m=1,m\neq l}^L(\mathbf{d}_l^T\mathbf{d}_m)^2$.

First, consider the K even case. By *Theorem 1*, part 1), we obtain $\mathrm{TSC}(\mathcal{S}) \geq K^2/L + 2|C(\{\mathbf{d}_1,\mathbf{d}_2,\ldots,\mathbf{d}_L\})|\left(2|1/\sqrt{L}|^2\right)^2 = K^2/L + (8/L^2)|C(\{\mathbf{d}_1,\mathbf{d}_2,\ldots,\mathbf{d}_L\})|$, and then, by *Theorem 1*, part 2)

$$\mathrm{TSC}(\mathcal{S}) \geq \begin{cases} \frac{K^2}{L}, & K\equiv 0 \pmod 4 \\ \frac{K^2}{L} + 2\frac{L-2}{L}, & K\equiv 2 \pmod 4 \\ & \text{and } L\equiv 0 \pmod 2 \\ \frac{K^2}{L} + 2\left(\frac{L-1}{L}\right)^2, & K\equiv 2 \pmod 4 \\ & \text{and } L\equiv 1 \pmod 2. \end{cases} \quad (6)$$

Next, we consider the K odd case. From *Theorem 1*, part 1), $\mathrm{TSC}(\mathcal{S}) \geq K^2/L + 2|C(\{\mathbf{d}_1,\mathbf{d}_2,\ldots,\mathbf{d}_L\})|(|1/\sqrt{L}|^2)^2 = K^2/L + (2/L^2)|C(\{\mathbf{d}_1,\mathbf{d}_2,\ldots,\mathbf{d}_L\})|$, and from *Theorem 1*, part 2), we conclude

$$\mathrm{TSC}(\mathcal{S}) \geq \frac{K^2}{L} + \frac{L-1}{L}, \qquad K \equiv 1 \pmod 2. \quad (7)$$

Equations (6) and (7) define the new bounds on the TSC of binary antipodal signature sets for overloaded $(K \geq L)$ systems and are summarized in Table II. Overloaded Gold signature sets

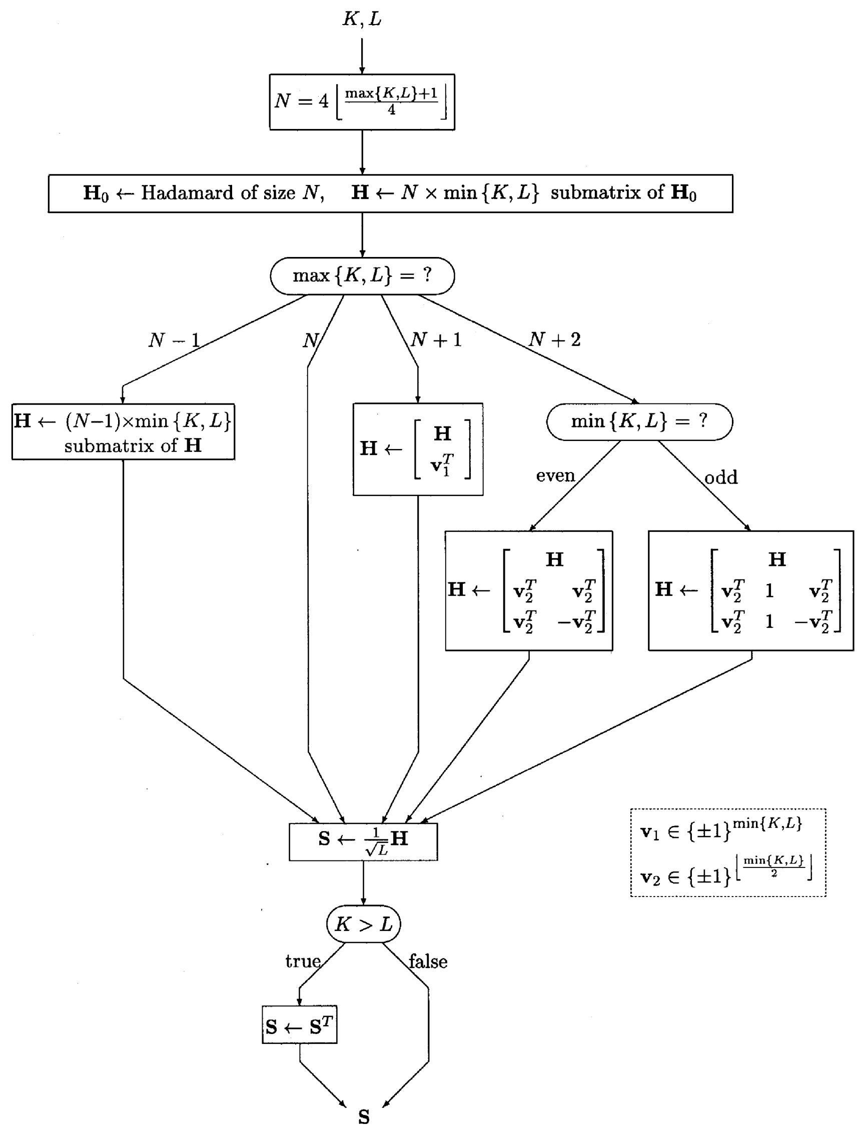

Fig. 1. Optimum binary antipodal signature set design procedure.

[10] with $K = L+1$ or $K = L+2$, $L = 2^p - 1$, $p = 5, 6, \ldots$, meet with equality the corresponding bound in (6) or (7) and, hence, are minimum-TSC optimum.[4]

II. Design of Minimum TSC Binary Antipodal Signature Sets

The following proposition identifies a sufficient condition under which the new bounds of Tables I and II become tight.

Proposition 1 (Conditions for Tightness of the TSC Bounds): Set $N \stackrel{\Delta}{=} 4\lfloor(\max\{K,L\}+1)/4\rfloor$. If $N \geq \min\{K,L\}$ and there exists a Hadamard matrix[5] of size N, then there exists a binary antipodal signature set $\mathcal{S} = \{\mathbf{s}_i\}_{i=1}^{K}$, $\mathbf{s}_i \in \{\pm 1/\sqrt{L}\}^L$, $i = 1, 2, \ldots, K$, that achieves equality in the corresponding TSC bound given by Table I or II. □

Fig. 1 acts as a proof-by-construction of *Proposition 1*. It presents in the form of a flow chart simple algorithms based on Hadamard matrix transformations for the design of optimum binary signature sets that achieve the TSC bound for both underloaded and overloaded systems. Our algorithmic exceptions are 1) $L = K \equiv 1 \,(\mathrm{mod}\ 4)$; 2) $L = K \equiv 2 \,(\mathrm{mod}\ 4)$; 3) $L+1 = K \equiv 2 \,(\mathrm{mod}\ 4)$; and 4) $K+1 = L \equiv 2 \,(\mathrm{mod}\ 4)$. A brief discussion on the design procedure follows.

For an underloaded system $(K \leq L)$, we have $N = 4\lfloor(L+1)/4\rfloor$ and $N \geq K$. By inspection, we observe that L takes one of the following four values: $L = N-1$, $L = N$, $L = N+1$, or $L = N+2$. Therefore, we need to design an optimum signature set, that is, a set that achieves the corresponding bound on TSC, for all four of the above cases (see Fig. 1). It is interesting to note that the design case $L = N-1$, $K \leq L$, in Fig. 1 covers as a proper subset the Gold signature sets [10] with $K \leq L$, $L = 2^p - 1$, $p = 5, 6, \ldots$, as well as the signature sets obtained by $K \leq L$ cyclic shifts of an m-sequence of

[4]Gold sets with $K = L+1$, $L = 2^p - 1$, $p = 5, 6, \ldots$, achieve the TSC bound K^2/L in (6), which of course coincides with the Welch bound for real-valued signatures. Therefore, the minimum-TSC optimality of overloaded-by-one Gold sets was already known [8]. (7) establishes the optimality of overloaded-by-two $(K = L+2)$ Gold sets, too.

[5]We recall that a Hadamard matrix of size N is an $N \times N$ matrix $\mathbf{A}$ with elements the real numbers $+1, -1$ and mutually orthogonal columns, $\mathbf{A}^T\mathbf{A} = N\mathbf{I}_{N\times N}$. A necessary condition for a Hadamard matrix to exist is that its size is a multiple of four, except for the trivial cases of size one or two.

length $L = 2^p - 1$, $p = 3, 4, \ldots$ [11]. Also, the design case $L = N$, $K \leq L$, in Fig. 1 covers as a proper subset the familiar Rademacher–Walsh orthogonal codes [12], [13] where $L = 2^p$, $p = 2, 3, \ldots$, and $K \leq L$, used in current CDMA technology [14] (direct Hadamard matrix design for this case where L is a multiple of four was suggested earlier in [2]).

For an overloaded system $(K \geq L)$, we have $N = 4\lfloor (K+1)/4 \rfloor$ and $N \geq L$. By inspection, we observe that K takes one of the following four values: $K = N-1$, $K = N$, $K = N+1$, or $K = N+2$. Therefore, we need to design an optimum signature set for each one of these four cases (see Fig. 1).

For each one of the above design cases for both underloaded and overloaded systems, we can show that $\mathrm{TSC}(\mathcal{S})$ is exactly equal to the corresponding new bound in Tables I or II. Therefore, our binary signature set design in Fig. 1 is minimum-TSC optimum.

III. Comments, Conclusions, and Examples

We derived new bounds on the TSC of binary antipodal signature sets for both underloaded and overloaded CDMA systems (summarized in Table I and Table II, respectively) and we identified sufficient conditions on the values of K (number of users) and L (processing gain) which guarantee that the corresponding new bounds on the TSC are tight. Our design of the optimum signature sets (and the tightness of the TSC bounds as it is described in *Proposition 1*) depends on the existence of a Hadamard matrix of size $N = 4\lfloor (\max\{K, L\} + 1)/4 \rfloor$. Assuming that in CDMA applications, values of K and L greater than 256 are not of much practical interest at present, we can mention that Hadamard matrices are known for all multiples of four less than or equal to 256 [15]. We conclude that the only pairs of values of K and L in $\{1, 2, \ldots, 256\}$ for which we cannot guarantee that the new bounds are tight (or, alternatively, we do not have a design method for constructing optimum sets) are the ones covered by the following cases: $L = K \equiv 1 \pmod 4$, $L = K \equiv 2 \pmod 4$, $L + 1 = K \equiv 2 \pmod 4$, and $K + 1 = L \equiv 2 \pmod 4$. It is interesting to note that these four combinations constitute a small percentage (0.38%) among all possible combinations of K and L in $\{1, 2, \ldots, 256\}$. We also note that as users enter or leave the system, the optimum signature set does not have to be redesigned unless we operate in an overloaded environment, and the change in the number of active users K dictates a switch to a new initial Hadamard signature set generation matrix $\mathbf{H}_0$. An example of the optimum binary signature set for an overloaded CDMA system with processing gain $L = 18$ and $K = 34$ users is given by the 18×34 signature matrix $\mathbf{S}$ in Fig. 2. This optimum set was obtained directly by the design procedure of Fig. 1.

$$\mathbf{S} = \begin{bmatrix}
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\end{bmatrix}$$

Fig. 2. Optimum binary antipodal signature set for an overloaded system with processing gain $L = 18$ and $K = 34$ users.

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