Supervised Phase Correction of Blind Space–Time DS-CDMA Channel Estimates

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***Abstract*—Blind channel-estimation algorithms return phase-ambiguous estimates. From a receiver design point of view, the phase-ambiguity problem can be by-passed by differential modulation and detection at the expense of a well-known performance loss, in comparison with direct modulation and coherent detection schemes. An alternative approach is followed in this paper. A theoretical minimum mean-square error phase-estimation criterion leads to a supervised phase-recovery procedure that directly corrects the phase of arbitrary linear filter receivers through a simple closed-form projection operation. Conveniently, any known blind channel-estimation algorithm can be used to provide the initial phase-ambiguous estimate. The presentation is given in the context of adaptive space–time receiver designs for binary phase-shift keying direct-sequence code-division-multiple-access antenna array systems. Numerical and simulation studies support the theoretical developments and show that effective phase correction and multiple-access interference suppression can be achieved with about 2% pilot signaling.**

***Index Terms*—Adaptive arrays, antenna arrays, code-division multiple access (CDMA), communication channels, differential phase-shift keying (DPSK), mean-square error (MSE) methods, phase estimation.**

I. Introduction

IN WIRELESS communications systems, multipath signal propagation phenomena are usually modeled at the baseband received signal as a convolution operation with a complex channel impulse response that induces a time delay and a multiplicative disturbance per path. Knowledge of the channel impulse response vector within a positive scalar ambiguity is necessary for the formulation of the familiar RAKE matched-filter receiver [1], [2]. For direct-sequence code-division multiple-access (DS-CDMA) communications, which is of particular interest to this work, the RAKE filter is simply the superposition of the appropriately shifted versions of the signature waveform of the user of interest multiplied by the corresponding channel coefficients [3]. When the channel coefficients are assumed random in an effort to model fading phenomena, the RAKE receiver has to be estimated adaptively based on an input data record size that conforms with the channel coherence time [2]. Unfortunately, second-order criteria for blind estimation of the channel coefficients [4]–[12] introduce a global phase ambiguity to the vector coefficient estimate. As a result, coherent receiver designs that rely *only* on a blind channel estimate cannot be pursued.

The concept of phase recovery was given some consideration in the past [13]–[15], primarily in the context of blind decision-directed phase tracking. Certainly, blind decision-directed phase tracking requires correct initialization and theoretically still suffers from an overall phase ambiguity that translates to symbol-detection ambiguity. In [16]–[25], differential phase-shift keying (DPSK) modulation and detection was considered for the design of DS-CDMA communications systems that avoid any form of channel phase dependence. DPSK communications are well known to incur a certain performance loss in comparison with their PSK counterparts [2]. Alternatively, in this paper, we consider plain binary PSK (BPSK) transmissions over multipath fading channels for DS-CDMA systems equipped with adaptive antenna arrays. We propose to use a very short pilot information bit sequence to recover the phase of the blind space–time (ST) channel estimate in the mean-square (MS) sense. The *supervised* MS-optimum phase-correction procedure is established for various linear ST receiver designs of interest, and it is shown that it always takes the form of a simple projection operation. An important characteristic of the algorithm is that the phase of the (possibly adaptive) receiver structure can be directly adjusted, without the need for receiver re-evaluation under the corrected channel-phase estimate. Moreover, the proposed phase-correction procedure is applicable to any blind channel-estimation method. For the purposes of this presentation, we choose to modify and generalize the subspace-based channel-estimation procedure in [6] to cover antenna array ST processing. In this context, the subspace *blind* ST channel-estimation algorithm provides us with a phase-ambiguous estimate of the RAKE vector, while the MS-optimum *supervised* phase-recovery procedure directly corrects the phase of any linear filter that is based on the ST RAKE vector estimate.

In terms of active ST multiple-access interference (MAI) suppression, we consider adaptive linear filter receivers such as the sample-matrix-inversion minimum-variance-distortionless-response (SMI-MVDR) filter [26]–[31] and the auxiliary-vector (AV) filter sequence [3], [16], [32]–[34]. Numerical and simulation studies included in this paper that involve blind subspace ST channel estimation support the theoretical MS-optimum phase-recovery developments. Indeed, the studies show that a very short pilot sequence (consisting of 4 or 5 bits only) is sufficient for the proposed phase-recovery algorithm to closely

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match the performance of receiver designs that assume perfect knowledge of the channel both in amplitude and phase.

This paper is organized as follows. In Section II, we introduce our notation and we present the received signal model under consideration. The core theoretical concepts of supervised MS-optimum phase recovery are developed in Section III. Estimation issues, including ST subspace channel estimation, are discussed in Section IV. Section V is devoted to numerical and simulation studies. A few conclusions are drawn in Section VI.

II. Signal Model

We consider mobile-to-base transmissions of K DS-CDMA users simultaneous in time and frequency with spreading gain L. The transmissions take place over multipath fading additive white Gaussian noise (AWGN) channels. Fading is modeled by multiplicative complex Gaussian random variables (Rayleigh distributed amplitude and uniformly distributed phase). The multipath fading channels are modeled by tapped-delay lines with independent fading per path. The received signal is collected by a narrowband antenna array of M elements. For illustration purposes, we consider uniform linear arrays. Identical fading is assumed to be experienced by all antenna elements for each path of each user signal (no antenna diversity). Details and notation are given below.

The contribution of the kth user, $k = 0, \ldots, K-1$, to the *transmitted* signal is denoted by

$$u_k(t) = \sum_i b_k(i)\sqrt{E_k} s_k(t - iT) e^{j(2\pi f_c t + \phi_k)} \quad (1)$$

where $b_k(i) \in \{-1, 1\}$ is the ith transmitted data (information) bit, E_k denotes energy, ϕ_k is the carrier phase with carrier frequency f_c, and $s_k(t)$ is the user signature waveform given by

$$s_k(t) = \sum_{l=0}^{L-1} d_k[l]\psi(t - lT_c) \quad (2)$$

where $d_k[l] \in \{-1, 1\}$ is the lth bit of the spreading sequence of the kth user, $\psi(t)$ is the chip waveform of unit energy, T_c is the chip period, and if T in (1) denotes the information bit duration, then $T/T_c = L$ is the spreading processing gain.

After multipath fading channel "processing," the total signal due to all users received at the input of a narrowband uniform linear array of M elements is given by

$$\mathbf{x}_c(t) = \sum_{k=0}^{K-1}\sum_{n=0}^{N-1} \alpha_{k,n} u_k\left(t - \frac{n}{B} - \tau_k\right)\mathbf{a}_{k,n} + \mathbf{n}(t) \quad (3)$$

where N is the total number of resolvable multipaths (without loss of generality, the number of resolvable multipaths is assumed to be the same for all users) and $\alpha_{k,n}, k = 0, \ldots, K-1, n = 0, \ldots, N-1$, are independent zero-mean complex Gaussian random variables that model the fading phenomena and are assumed to remain constant over several bit intervals. We recall that, in fact, measurements have shown that mobiles on foot operating at $f_c = 900$ MHz induce a typical fading rate of about 4.5 Hz, while fast-moving vehicles may cause fading rates as high as 70 Hz [35]. Moreover, τ_k in (3) denotes the relative transmission delay of user k with respect to user 0 with $\tau_0 = 0$, and with $\mathbf{x}_c(t)$ bandlimited to $B = 1/T_c$, the tapped-delay line channel model has taps spaced at chip intervals T_c. The $M \times 1$ array response vector $\mathbf{a}_{k,n}$ for the nth path of the kth user signal is defined by

$$\mathbf{a}_{k,n}[m] = e^{j2\pi(m-1)\frac{\sin\theta_{k,n} d}{\lambda}}, \quad m = 1, \ldots, M \quad (4)$$

where $\theta_{k,n}$ identifies the angle of arrival of the nth path signal from the kth user, λ is the carrier wavelength, and d is the element spacing of the antenna array (usually $d = \lambda/2$). Finally, $\mathbf{n}(t)$ in (3) denotes an M-dimensional complex Gaussian noise process that is assumed white both in time and space.

After carrier demodulation

$$\mathbf{x}(t) = \sum_i \sum_{k=0}^{K-1} b_k(i) \times \sum_{n=0}^{N-1} c_{k,n} s_k\left(t - iT - \frac{n}{B} - \tau_k\right)\mathbf{a}_{k,n} + \mathbf{n}(t) \quad (5)$$

where $c_{k,n} = \sqrt{E_k}\alpha_{k,n}e^{-j(2\pi f_c(n/B)+\gamma_k)}$ with $\gamma_k = 2\pi f_c\tau_k - \phi_k$ being the total carrier phase absorbed into the channel coefficient. Assuming synchronization at the reference antenna element $(m = 1)$ with the signal of the user of interest, for example user 0, chip-matched filtering and sampling of $\mathbf{x}(t)$ at the chip rate $1/T_c$ over the multipath extended period of $L+N-1$ chips prepares the data for one-shot detection of the ith information bit of interest $b_0(i)$. We visualize the collected ST data in the form of an $M \times (L+N-1)$ matrix

$$\mathbf{X}_{M\times(L+N-1)} = [\mathbf{x}(0) \quad \mathbf{x}(T_c) \quad \ldots \quad \mathbf{x}((L+N-2)T_c)]. \quad (6)$$

To avoid cumbersome 2-D filtering operations and notation, we decide at this time to "vectorize" $\mathbf{X}_{M\times(L+N-1)}$ by sequencing all matrix columns in the form of a vector

$$\mathbf{x}_{M(L+N-1)\times 1} = \text{Vec}\{\mathbf{X}_{M\times(L+N-1)}\}. \quad (7)$$

From now on, $\mathbf{x}$ denotes the joint ST data in the $\mathcal{C}^{M(L+N-1)}$ complex vector domain.

The cornerstone for any form of ST filtering is the ST RAKE filter that we define for user 0 as the superposition of shifted versions of the ST matched filter multiplied by the corresponding channel coefficients

$$\mathbf{v}_0 \triangleq \sum_{n=0}^{N-1} c_{0,n}\left[\underbrace{0\ldots0}_{n} d_0[0]\ldots d_0[L-1] \underbrace{0\ldots0}_{N-n-1}\right]^T \odot \mathbf{a}_{0,n} \quad (8)$$

where $\odot$ denotes the Kronecker tensor product. Theoretically, the ST RAKE receiver $\mathbf{v}_0$ is also given by

$$\mathbf{v}_0 = E_{b_0}\{\mathbf{x}b_0^*\} \quad (9)$$

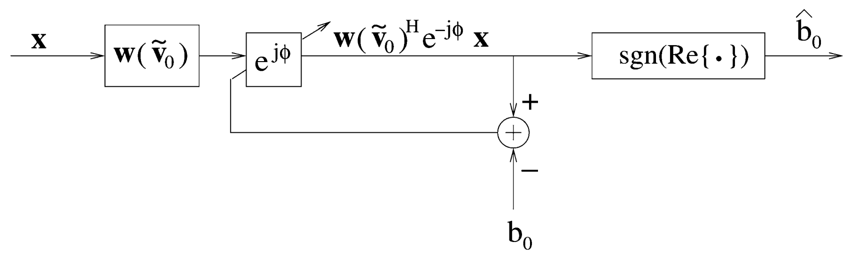

Fig. 1. Phase correction for the ST linear filter $\mathbf{w}(\tilde{\mathbf{v}}_0)$.

where the statistical expectation operation $E\{\cdot\}$ is taken with respect to b_0 only.

The ST RAKE receiver of (8) or (9) is a function of the binary signature vector (spreading sequence) of the user of interest (user 0), the channel coefficients $c_{0,0}, c_{0,1}, \ldots, c_{0,N-1}$, and the corresponding angles of arrival $\theta_{0,0}, \theta_{0,1}, \ldots, \theta_{0,N-1}$. While the spreading sequence is assumed to be known to the receiver, the channel coefficients and the angles of arrival are, in general, unknown. What we are concerned with in this paper are the consequences of blind estimation of $\mathbf{v}_0$ from a given finite-size record of input data. Since blind channel-estimation methods induce phase ambiguity, we propose to recover the phase of the estimate using a short pilot bit sequence.

III. MS-Optimum Phase Recovery

In this section, we consider the recovery (correction) of the phase of ST linear filters when the ST RAKE vector $\mathbf{v}_0$ is known within a phase ambiguity.

Let $\tilde{\mathbf{v}}_0$ denote a phase-ambiguous version of $\mathbf{v}_0$, i.e.,

$$\tilde{\mathbf{v}}_0 e^{j\phi} = \mathbf{v}_0 \tag{10}$$

where ϕ is the unknown phase. We consider the class of linear filters $\mathbf{w} \in \mathcal{C}^{M(L+N-1)}$ that are functions of the ST RAKE vector $\mathbf{v}_0$ and share the following property:

$$\mathbf{w}(\mathbf{v}_0) = \mathbf{w}(\tilde{\mathbf{v}}_0) e^{j\phi}. \tag{11}$$

Such filters are of prime importance for the design of ST DS-CDMA receivers, and they include:

1) the ST RAKE filter itself $\mathbf{v}_0$;
2) the ST MVDR filter

$$\mathbf{w}_{\mathrm{MVDR}} = \frac{\mathbf{R}^{-1}\mathbf{v}_0}{\mathbf{v}_0^H \mathbf{R}^{-1} \mathbf{v}_0} \tag{12}$$

where $\mathbf{R} = E\{\mathbf{x}\mathbf{x}^H\}$;

3) the AV sequence of ST filters $\mathbf{w}_n$ defined by the following recursion [3], [16], [33], [34]:

$$\mathbf{w}_0 = \mathbf{v}_0, \tag{13}$$

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mu_{n+1}\mathbf{g}_{n+1}, \quad n = 0, 1, 2, \ldots \tag{14}$$

where

$$\mathbf{g}_{n+1} = \left(\mathbf{I} - \frac{\mathbf{v}_0 \mathbf{v}_0^H}{\|\mathbf{v}_0\|^2}\right)\mathbf{R}\mathbf{w}_n \tag{15}$$

$$\mu_{n+1} = \frac{\mathbf{g}_{n+1}^H \mathbf{R}\mathbf{w}_n}{\mathbf{g}_{n+1}^H \mathbf{R}\mathbf{g}_{n+1}}. \tag{16}$$

In terms of notation, $\mathbf{x}^H$ denotes the Hermitian of $\mathbf{x}$; in (15), $\mathbf{I}$ denotes the $M(L+N-1) \times M(L+N-1)$ identity matrix.

As seen by (11), for this class of filters, the phase ambiguity of $\tilde{\mathbf{v}}_0$ leads to a phase-ambiguous linear filter $\mathbf{w}(\tilde{\mathbf{v}}_0)$. Given $\tilde{\mathbf{v}}_0$, we attempt to correct the phase of $\mathbf{w}(\tilde{\mathbf{v}}_0)$ as follows (Fig. 1). The selection criterion for the phase correction ϕ that we propose in this paper is the minimization of the mean-square error (MSE) between the phase-corrected ST filter processed data $[\mathbf{w}(\tilde{\mathbf{v}}_0)e^{j\phi}]^H\mathbf{x}$ and the desired information bit b_0

$$\hat{\phi} = \arg\min_{\phi} E\{|[\mathbf{w}(\tilde{\mathbf{v}}_0)e^{j\phi}]^H\mathbf{x} - b_0|^2\}, \quad \phi \in [-\pi, \pi). \tag{17}$$

The following proposition identifies the optimum phase correction according to our criterion.

Proposition 1: The phase correction

$$\hat{\phi} = \text{angle}\,\{\mathbf{w}(\tilde{\mathbf{v}}_0)^H E\{\mathbf{x}b_0^*\}\} \tag{18}$$

minimizes the MSE between the phase-corrected ST filter processed data $[\mathbf{w}(\tilde{\mathbf{v}}_0)e^{j\phi}]^H\mathbf{x}$ and the desired information bit b_0.□

Essentially, *Proposition 1* suggests projecting the phase-ambiguous $\mathbf{w}(\tilde{\mathbf{v}}_0)$ filter onto the ideal ST RAKE filter $\mathbf{v}_0 = E\{\mathbf{x}b_0^*\}$. We understand, of course, that $E\{\mathbf{x}b_0^*\}$ is not known. If a pilot information symbol sequence of length P is available, the expectation $E\{\mathbf{x}b_0^*\}$ can be sample-average estimated by $(1/P)\sum_{i=1}^{P}\mathbf{x}_i b_0^*(i)$.[1] Then, for example, the phase-correction estimate for the MVDR filter $\mathbf{w}(\tilde{\mathbf{v}}_0) = (\mathbf{R}^{-1}\tilde{\mathbf{v}}_0)/(\tilde{\mathbf{v}}_0^H\mathbf{R}^{-1}\tilde{\mathbf{v}}_0)$ becomes

$$\hat{\phi} = \text{angle}\left\{\tilde{\mathbf{v}}_0^H \mathbf{R}^{-1}\sum_{i=1}^{P}\mathbf{x}_i b_0^*(i)\right\}. \tag{19}$$

[1]Alternatively, one might consider applying the phase-ambiguous filter $\mathbf{w}(\tilde{\mathbf{v}}_0)$ at the points where the known pilot symbols occur, and directly compare the filter output with the known pilot symbol values. Then, the least-squares (LS) phase-correction estimate that minimizes the LS metric $\sum_{i=1}^{P}|(\mathbf{w}(\tilde{\mathbf{v}}_0)e^{j\phi})^H\mathbf{x}_i - b_0(i)|^2$, is given by $\hat{\phi} = \text{angle}\,\{\mathbf{w}(\tilde{\mathbf{v}}_0)^H\sum_{i=1}^{P}\mathbf{x}_i b_0^*(i)\}$. Therefore, LS estimation of ϕ coincides with MS optimum phase recovery when $E\{\mathbf{x}_i b_0^*(i)\}$ in (18) is replaced by its sample-average estimate.

It can be shown that $\hat{\phi}$ in (19) is also the maximum-likelihood (ML) estimate of ϕ when a data record $\{(\mathbf{x}_1, b_0(1)), (\mathbf{x}_2, b_0(2)), \ldots, (\mathbf{x}_P, b_0(P))\}$ is available and the interference-plus-noise contribution to the received vector $\mathbf{x}_i$ is modeled as a colored Gaussian vector. This is in sharp contrast to previous work [2], [36] that ignored the interference (or, equivalently, treated the interference-plus-noise contribution as a white Gaussian vector), leading to the ML estimate

$$\hat{\phi} = \text{angle}\left\{\tilde{\mathbf{v}}_0^H \sum_{i=1}^{P} \mathbf{x}_i b_0^*(i)\right\}. \tag{20}$$

Apart from the expectation $E\{\mathbf{x}b_0^*\}$, we recall that in reality, $\tilde{\mathbf{v}}_0$ and the input autocorrelation matrix $\mathbf{R}$ (needed for the calculation of the MVDR and AV filters) are also unknown. In the next section, we study in some detail the problem of estimating $\tilde{\mathbf{v}}_0, E\{\mathbf{x}b_0^*\}$, and $\mathbf{R}$ from a finite-size data record. It is important to observe that if (11) holds true and $\mathbf{w}(\mathbf{v}_0)$ is either the ST RAKE, ST MVDR, or ST AV filter, then angle $\{\mathbf{w}(\tilde{\mathbf{v}}_0)^H E\{\mathbf{x}b_0^*\}\} =$ angle $\{\tilde{\mathbf{v}}_0^H E\{\mathbf{x}b_0^*\}\}$. However, this equality no longer holds when $\tilde{\mathbf{v}}_0, E\{\mathbf{x}b_0^*\}$, and $\mathbf{R}$ are estimated. In the latter case, it is prudent to recover *directly* the phase of the filter estimate $\mathbf{w}(\hat{\tilde{\mathbf{v}}}_0)$ as suggested by (18).

IV. Estimation Considerations

Assume that a finite ST input data record of size J is available: $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_J$. To estimate the channel coefficients $\vec{\mathbf{c}}_0 = [c_{0,0}, c_{0,1}, \ldots, c_{0,N-1}]^T$ and the angles of arrival $\vec{\theta}_0 = [\theta_{0,0}, \theta_{0,1}, \ldots, \theta_{0,N-1}]^T$ for the user of interest, we resort to a subspace-based estimation procedure. This procedure can be viewed as a generalization of the algorithm in [6] for an antenna-array setup. An additional modification is introduced that increases the rank of the noise subspace, as discussed below.

Under the assumption that $N \leq (L/2) + 1$ (a rather reasonable assumption for DS-CDMA communications), the ith received ST data vector $\mathbf{x}_i$ can be expressed as

$$\mathbf{x}_i = \sum_{k=0}^{K-1} [b_k(i)\mathbf{v}_k + b_k(i-1)\mathbf{v}_k^- + b_k(i+1)\mathbf{v}_k^+] + \mathbf{n}_i, \quad i = 1, 2, \ldots, J \tag{21}$$

where $\mathbf{v}_0$ is the ST RAKE vector of the user of interest in (8) and (9), $\mathbf{v}_k \triangleq E_{b_k(i)}\{\mathbf{x}_i b_k^*(i)\}, k = 1, 2, \ldots, K-1$, and $\mathbf{v}_k^- \triangleq E_{b_k(i-1)}\{\mathbf{x}_i b_k^*(i-1)\}, \mathbf{v}_k^+ \triangleq E_{b_k(i+1)}\{\mathbf{x}_i b_k^*(i+1)\}$ for $k = 0, 1, \ldots, K-1$. The three vector terms of the user of interest, $\mathbf{v}_0, \mathbf{v}_0^-$, and $\mathbf{v}_0^+$, are always present in (21). Each interferer, $k = 1, \ldots, K-1$, contributes to (21) two or three additional vector terms ($\mathbf{v}_k$ and $\mathbf{v}_k^-$ or $\mathbf{v}_k^+$ or both), depending on the exact value of the relative delay τ_k with respect to user 0. Therefore, the possible values of the rank r_s of the *signal subspace* of $\mathbf{x}_i$ are $2K+1 \leq r_s \leq 3K$. The rank r_s can be controlled by data truncation. One-sided-only truncation of $\mathbf{x}_i$ eliminates either the $\mathbf{v}_0^-$ or $\mathbf{v}_0^+$ component, and the signal-subspace rank is reduced by at least one, $2K \leq r_s \leq 3K-1$. We can further reduce r_s by truncating both sides of $\mathbf{x}_i$ as shown in Fig. 2. Then the *truncated received vector* $\mathbf{x}_i^{\text{tr}}$ contains only $\mathbf{v}_0$ (two-sided truncation of $\mathbf{x}_i$ eliminates intersymbol interference (ISI) for user 0) and at most two vectors per interferer, $\mathbf{v}_k$ and $\mathbf{v}_k^-$ or $\mathbf{v}_k^+, k = 1, \ldots, K-1$. Thus, $K \leq r_s \leq 2K-1$. In summary, the possible values of r_s depending on the data formation of choice are as follows.

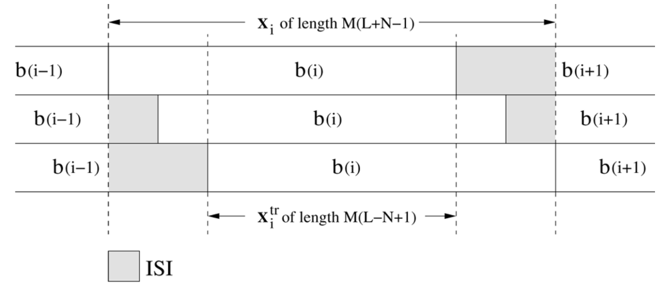

Fig. 2. Data collection and ISI trimming.

1) No truncation:
Data dimension $= M(L+N-1), 2K+1 \leq r_s \leq 3K$.
2) One-sided truncation:
Data dimension $= ML, 2K \leq r_s \leq 3K-1$.
3) Two-sided truncation:
Data dimension $= M(L-N+1), K \leq r_s \leq 2K-1$.

The ISI terms $b_k(i-1)\mathbf{v}_k^-$ and $b_k(i+1)\mathbf{v}_k^+, k = 0, 1, \ldots, K-1$ in (21) are characterized by low energy per received vector (by definition, many coordinates of $\mathbf{v}_k^-$ and $\mathbf{v}_k^+$ are zero). Hence, certain signal eigenvalues are expected to be small and close to the noise eigenvalues.[2] To assist the subspace ST channel-estimation procedure and have the maximum possible *guaranteed minimum rank* of the *noise subspace* of $M(L-N+1) - (2K-1)$, we truncate $\mathbf{x}$ from both sides (Case 3), and we form the "truncated" received vector $\mathbf{x}^{\text{tr}}$ of length $M(L-N+1)$ as follows:

$$\mathbf{x}^{\text{tr}} = \begin{bmatrix} \mathbf{x}((N-1)T_c) \\ \mathbf{x}(NT_c) \\ \vdots \\ \mathbf{x}((L-1)T_c) \end{bmatrix}. \tag{22}$$

Then, with respect to the ith information bit of user 0, $\mathbf{x}_i^{\text{tr}}$ can be expressed as

$$\mathbf{x}_i^{\text{tr}} = b_0(i)\mathbf{A}_0\mathbf{B}(\vec{\theta}_0)\vec{\mathbf{c}}_0 + \text{MAI}_i + \mathbf{n}_i^{\text{tr}} \tag{23}$$

where MAI_i accounts comprehensively for ST MAI of rank $r_s - 1$, $\mathbf{B}(\vec{\theta}_0)$ is a block diagonal matrix of the form $\mathbf{B}(\vec{\theta}_0) \triangleq \text{diag}(\mathbf{a}_{0,0}, \mathbf{a}_{0,1}, \ldots, \mathbf{a}_{0,N-1})$, and $\mathbf{A}_0 = \mathbf{A}_0^s \odot \mathbf{I}_M$. The symbol $\odot$ denotes the Kronecker tensor product, $\mathbf{I}_M$ is the $M \times M$ identity matrix, and

$$\mathbf{A}_0^s \triangleq \begin{bmatrix} d_0[N-1] & d_0[N-2] & \ldots & d_0[0] \\ d_0[N] & d_0[N-1] & \ldots & d_0[1] \\ \vdots & \vdots & & \vdots \\ d_0[L-1] & d_0[L-2] & \ldots & d_0[L-N] \end{bmatrix}. \tag{24}$$

Let $\mathbf{R}_{\text{tr}} = E\{\mathbf{x}^{\text{tr}}\mathbf{x}^{\text{tr}^H}\}$ be the autocorrelation matrix of $\mathbf{x}^{\text{tr}}$. If $\mathbf{R}_{\text{tr}} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H$ represents the eigendecomposition of $\mathbf{R}_{\text{tr}}$, where the columns of $\mathbf{Q}$ are the eigenvectors of $\mathbf{R}_{\text{tr}}$ and $\mathbf{\Lambda}$ is

[2]The problem case of hard-to-separate signal and noise subspace estimates was studied recently in [37]–[39].

a diagonal matrix consisting of the eigenvalues of $\mathbf{R}_{\rm tr}$, then the eigenvectors that correspond to the $M(L-N+1)-(2K-1)$ smallest eigenvalues are all guaranteed to belong to the noise subspace. Let the matrix $\mathbf{U}_n$ of size $[M(L-N+1)]\times[M(L-N+1)-(2K-1)]$ consist of these $M(L-N+1)-(2K-1)$ "noise eigenvectors."

We select $\vec{\mathbf{c}}_0$ and $\vec{\theta}_0$ as the vectors that make the signal of user 0, $\mathbf{A}_0\mathbf{B}(\vec{\theta}_0)\vec{\mathbf{c}}_0$, orthogonal to the noise subspace [6], [40]

$$(\mathbf{A}_0\mathbf{B}(\vec{\theta}_0)\vec{\mathbf{c}}_0)^H\mathbf{U}_n=\mathbf{0},\quad \text{subject to } \|\vec{\mathbf{c}}_0\|=1. \tag{25}$$

The solution to this optimization criterion is given by the following proposition.

Proposition 2: The projection of the signal of user 0 to the $\mathbf{U}_n$ noise subspace becomes zero when $\vec{\theta}_0$ is chosen to make the matrix $\mathbf{M}_0(\vec{\theta}_0)\triangleq\mathbf{B}(\vec{\theta}_0)^H\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0\mathbf{B}(\vec{\theta}_0)$ singular, and $\vec{\mathbf{c}}_0$ is selected as the eigenvector that corresponds to the zero-eigenvalue of $\mathbf{M}_0(\vec{\theta}_0)$ for that specific choice of $\vec{\theta}_0$. □

It is straightforward to verify that the vectors $\vec{\theta}_0$ and $\vec{\mathbf{c}}_0$ identified by *Proposition 2* satisfy (25). However, they are not guaranteed to be the *true* ST channel-parameter vectors unless the specific $\vec{\theta}_0,\vec{\mathbf{c}}_0$ solution is unique. The following proposition identifies a sufficient condition for a unique solution in *Proposition 2*. The proof is given in the Appendix.

Proposition 3: The ST channel-parameter vectors $\vec{\theta}_0$ and $\vec{\mathbf{c}}_0$ are uniquely determined by *Proposition 2* if:

1) rank $(\mathbf{A}_0^s)=N$; and
2) $$\text{null}\,(\mathbf{U}_n^H)\bigcap\text{range}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta}))=\begin{cases}\text{range}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta}_0)\vec{\mathbf{c}}_0), & \vec{\theta}=\vec{\theta}_0\\ \{\mathbf{0}\}, & \vec{\theta}\neq\vec{\theta}_0.\end{cases}$$

□

The requirements of *Proposition 3* may be satisfied by appropriate design of the user signature set. Condition 1) requires $\mathbf{A}_0^s$ to be a full-column-rank matrix. If null $(\mathbf{U}_n^H)$ coincides with the signal subspace (that is, $r_s=2K-1$ and our matrix $\mathbf{U}_n$ covers the whole noise subspace), Condition 2) in essence requires that no linear combination of the (truncated) ST user signatures can form a valid (truncated) ST signature configuration for the user of interest, except for the true one.

It is interesting to observe that according to *Proposition 2*, the joint optimization of $\vec{\mathbf{c}}_0$ and $\vec{\theta}_0$ becomes disjoint. However, in the absence of a closed-form expression for the optimum $\vec{\theta}_0$, only numerical optimization of the vector $\vec{\theta}_0$ can be pursued at considerable computational cost. For this reason, we present an alternative approach by combining $\vec{\mathbf{c}}_0$ and $\vec{\theta}_0$ into one $MN\times 1$ vector $\mathbf{h}_0$

$$\mathbf{h}_0\triangleq\mathbf{B}(\vec{\theta}_0)\vec{\mathbf{c}}_0. \tag{26}$$

As before, we seek the vector $\mathbf{h}_0$ that makes the projection of the signal of the user of interest (user 0) onto the noise subspace equal to zero

$$(\mathbf{A}_0\mathbf{h}_0)^H\mathbf{U}_n=\mathbf{0},\quad \text{subject to } \|\mathbf{h}_0\|=1. \tag{27}$$

The solution to this selection criterion is given by the following proposition.

Proposition 4: The vector $\mathbf{h}_0$ that makes the projection of the signal of user 0 to the $\mathbf{U}_n$ noise subspace equal to zero, subject to the norm constraint $\|\mathbf{h}_0\|=1$, is the eigenvector that corresponds to the zero-eigenvalue of $\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0$. □

The following proposition identifies a necessary and sufficient condition for a unique solution. The proof is given in the Appendix.

Proposition 5: The ST channel parameter vector $\mathbf{h}_0$ is uniquely determined by *Proposition 4* if and only if:

1) rank $(\mathbf{A}_0^s)=N$; and
2) null $(\mathbf{U}_n^H)\bigcap$ range $(\mathbf{A}_0)=$ range $(\mathbf{A}_0\mathbf{h}_0)$.

□

Once again, the requirements of *Proposition 5* can be satisfied by appropriate design of the user signature set. Condition 1) implies that $\mathbf{A}_0^s$ is a full-column-rank matrix. If null $(\mathbf{U}_n^H)$ coincides with the signal subspace, Condition 2) implies that the signal subspace and the subspace spanned by the columns of $\mathbf{A}_0$ do not have any common elements, except for the true "truncated" signal vector of interest $\mathbf{A}_0\mathbf{B}(\vec{\theta}_0)\vec{\mathbf{c}}_0=\mathbf{A}_0\mathbf{h}_0$ (and, of course, all complex scalar multiples of it).

Proposition 4 relies on knowledge of the autocorrelation matrix $\mathbf{R}_{\rm tr}$. Since, in reality, $\mathbf{R}_{\rm tr}$ is not known, we form a sample-average estimate

$$\hat{\mathbf{R}}_{\rm tr}=\frac{1}{J}\sum_{i=1}^{J}\mathbf{x}_i^{\rm tr}\mathbf{x}_i^{{\rm tr}\,H} \tag{28}$$

based on the truncated J available ST input vectors $\mathbf{x}_i^{\rm tr}, i=1,2,\ldots,J$. Let $\hat{\mathbf{U}}_n$ be the matrix consisting of the "bottom" $M(L-N+1)-(2K-1)$ "noise eigenvectors" of $\hat{\mathbf{R}}_{\rm tr}$. The signal of user 0 is not guaranteed to be completely orthogonal to $\hat{\mathbf{U}}_n$. Therefore, we choose to seek the vector $\hat{\mathbf{h}}_0$ that minimizes the norm of the projection of the signal of user 0 onto the estimated noise subspace $\hat{\mathbf{U}}_n$

$$\hat{\mathbf{h}}_0=\arg\min_{\mathbf{h}_0}\|(\mathbf{A}_0\mathbf{h}_0)^H\hat{\mathbf{U}}_n\|,\quad \text{subject to } \|\hat{\mathbf{h}}_0\|=1. \tag{29}$$

The solution to this selection criterion is given by the following proposition.

Proposition 6: The vector $\hat{\mathbf{h}}_0$ that minimizes the projection of the signal of user 0 to the $\hat{\mathbf{U}}_n$ estimated noise subspace subject to the norm constraint $\|\hat{\mathbf{h}}_0\|=1$ is the eigenvector that corresponds to the minimum eigenvalue of $\mathbf{A}_0^H\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H\mathbf{A}_0$. □

After obtaining $\hat{\mathbf{h}}_0$, we may extract the desired vectors $\hat{\vec{\mathbf{c}}}_0$ and $\hat{\vec{\theta}}_0$ by applying LS fitting to $\hat{\mathbf{h}}_0$. Then, our estimate $\hat{\hat{\mathbf{v}}}_0$ is completely defined by (8).

As mentioned earlier, the criteria employed above in *Propositions 2, 4*, or *6* absorb the phase information, which means that the estimate $\hat{\hat{\mathbf{v}}}_0$ is phase-ambiguous. What remains to be done is phase correction of the overall receiver filter $\mathbf{w}(\hat{\hat{\mathbf{v}}}_0)$ as introduced in Section III, *Proposition 1*. If a pilot information bit sequence of length P is available, then the expectation $E\{\mathbf{x}b_0^*\}$ in (18) can be sample-average estimated by $(1/P)\sum_{i=1}^{P}\mathbf{x}_ib_0^*(i)$. Then, for example, the phase-corrected ST RAKE filter estimate is given by

$$\hat{\hat{\mathbf{v}}}_0e^{j\hat{\phi}},\quad \hat{\phi}=\text{angle}\left\{\hat{\hat{\mathbf{v}}}_0^H\left[\sum_{i=1}^{P}\mathbf{x}_ib_0^*(i)\right]\right\}. \tag{30}$$

If instead of plain ST RAKE filtering, we choose to actively suppress interference, then we may consider the MVDR [26]–[31] or AV [3], [16], [32]–[34] linear filters that are all defined as a function of $\mathbf{v}_0$ and the input autocorrelation matrix $\mathbf{R}$ (cf. Section III), $\mathbf{w}(\mathbf{v}_0, \mathbf{R})$. With respect to $\mathbf{R}$, we form a sample-average estimate from the same given input data record of size J that we used for ST channel estimation, but this time without data truncation

$$\hat{\mathbf{R}} = \frac{1}{J}\sum_{i=1}^{J}\mathbf{x}_i\mathbf{x}_i^H. \tag{31}$$

With respect to $\mathbf{v}_0$, we already have a phase-ambiguous estimate $\hat{\hat{\mathbf{v}}}_0$ at hand. Next, according to the discussion in Section III, we first generate the phase-ambiguous filter $\mathbf{w}(\hat{\hat{\mathbf{v}}}_0, \hat{\mathbf{R}})$ and then we correct its phase

$$\mathbf{w}(\hat{\hat{\mathbf{v}}}_0, \hat{\mathbf{R}})e^{j\hat{\phi}}, \quad \hat{\phi} = \text{angle}\left\{\mathbf{w}(\hat{\hat{\mathbf{v}}}_0, \hat{\mathbf{R}})^H\left[\sum_{i=1}^{P}\mathbf{x}_i b_0^*(i)\right]\right\}. \tag{32}$$

When J represents the packet size of a DS-CDMA system and P is the number of preamble (or midamble [41]) pilot information bits per packet, then the ratio P/J quantifies the wasted bandwidth due to the use of the pilot bit sequence. Ideally, for packet-rate adaptive receiver designs, both P/J and J are to be kept small (the latter is necessary to ensure that the user ST channels are nearly constant over the collected packet input data time period). It is known that AV filter designs cope well with small input data support J [3], [16], [32]–[34]. In addition, as we will see in the following section, a few pilot bits (of the order of 5 bits) are sufficient for effective recovery of the AV filter phase. As a numerical example, when the packet size is set at $J = 256$ and $P = 5$ is chosen, the wasted bandwidth is only $P/J \simeq 2\%$.

V. Numerical and Simulation Comparisons

We consider the DS-CDMA signal model in Section II for a system with $M = 4$ antenna elements and spreading gain $L = 31$. We compare the bit-error rate (BER) performance of the ST RAKE filter in (8), the ST MVDR filter in (12), and the ST AV filters in (14)–(16) for subspace-based ST channel estimation, as described in Section IV, and MS-optimum phase recovery per filter, as described in (32). We assume the presence of $K = 20$ active users. Each user signal experiences $N = 3$ independent Rayleigh fading paths with equal average received energy per path, and independent angles of arrival uniformly distributed in $(-\pi/2, \pi/2)$. The array interelement spacing is taken to be half-the-wavelength, and identical fading is assumed to be experienced by all antenna elements for each path of each user signal.

In Fig. 3, we fix the total signal-to-noise ratios (SNRs) of the interferers at $\text{SNR}_1 - \text{SNR}_4 = 7$ dB, $\text{SNR}_5 - \text{SNR}_8 = 8$ dB, $\text{SNR}_9 - \text{SNR}_{12} = 9.5$ dB, $\text{SNR}_{13} - \text{SNR}_{16} = 10.5$ dB, and $\text{SNR}_{17} - \text{SNR}_{19} = 12$ dB, and we set the total SNR of the user of interest at 10 dB. We assume perfect knowledge of the input autocorrelation matrix $\mathbf{R}$ (and of its data-truncated version $\mathbf{R}_{\text{tr}}$), and we plot as a function of the length of the pilot bit sequence P the BER induced by the ST RAKE filter and ST MVDR filter with the proposed direct filter-phase correction in (32). We compare with the performance of the ST RAKE and ST MVDR filters that incorporate the conventional channel-phase estimate of [2] and [36] given by (20). As a reference, we include the RAKE and MVDR receivers when perfect knowledge of the ST channel is available. Under plain RAKE filtering, conventional channel-phase estimation and the proposed filter-phase correction scheme coincide theoretically. This is not the case, of course, for interference suppressing MVDR filtering, as seen in Fig. 3. The results that we present therein are averages over 200 independently drawn multipath Rayleigh fading ST channels. We note the universal 1/2 BER value when no phase correction of the ST channel estimate is attempted $(P = 0)$, and the superiority of the filter-phase correction approach under MVDR filtering for $P \geq 1$.

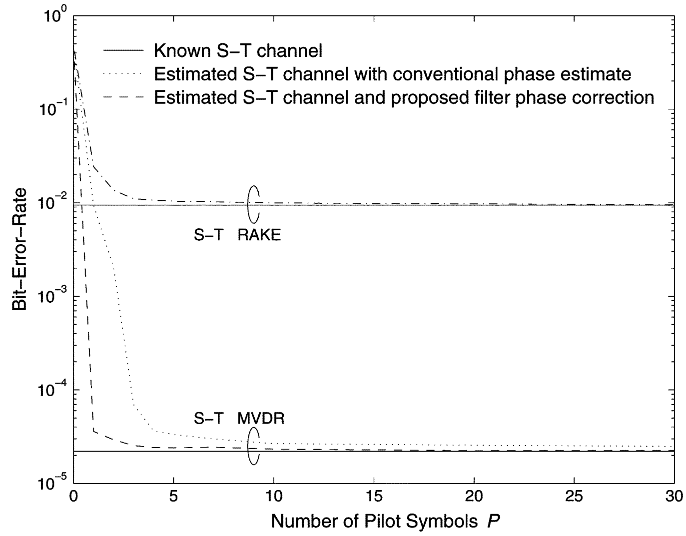

Fig. 3. BER versus pilot bit sequence length for the ST RAKE and MVDR filters. The total SNR of the user of interest is fixed at 10 dB. Perfect knowledge of the input autocorrelation matrix is assumed.

In Fig. 4, we assume that the input autocorrelation matrix $\mathbf{R}$ (and $\mathbf{R}_{\text{tr}}$) is unknown. We form a sample-average estimate $\hat{\mathbf{R}}$ (and $\hat{\mathbf{R}}_{\text{tr}}$) from a data record of size $J = 256$, and we plot the BER induced by the SMI-MVDR filter, the $\mathbf{w}_2$ AV filter (two auxiliary vectors), and the ST RAKE filter (zero auxiliary vectors) as a function of the length of the pilot bit sequence P. The results that we present are averages over 200 independently drawn multipath Rayleigh fading ST channels and 20 independent filter realizations per ST channel. As a reference, we include the SMI-MVDR, 2-AV, and RAKE receivers when perfect knowledge of the ST channel is available. As we see, 4 or 5 pilot bits are sufficient for effective recovery of the 2-AV filter phase. On a side note, as expected [32], the SMI-MVDR receiver suffers from severe data starvation (J is too small) and does not represent an acceptable solution.

In Fig. 5, we repeat the BER studies of Fig. 4 as a function of the total SNR of the user of interest when the sample support is fixed at $J = 256$ and $P = 5$ pilot bits are used. It appears that phase-corrected AV receiver designs based on subspace ST channel estimates form an appealing solution for antenna-array DS-CDMA systems.

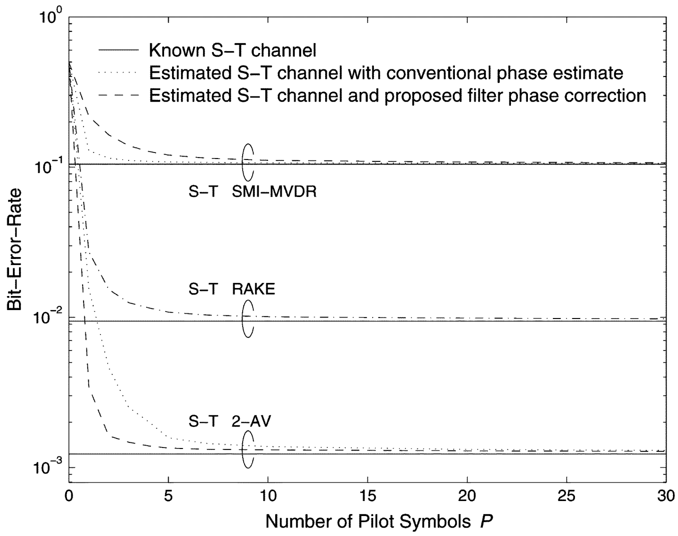

Fig. 4. BER versus pilot bit sequence length for the ST SMI-MVDR, RAKE, and 2-AV filters. The total SNR of the user of interest is fixed at 10 dB. Sample support for the estimation of the input autocorrelation matrix is $J = 256$.

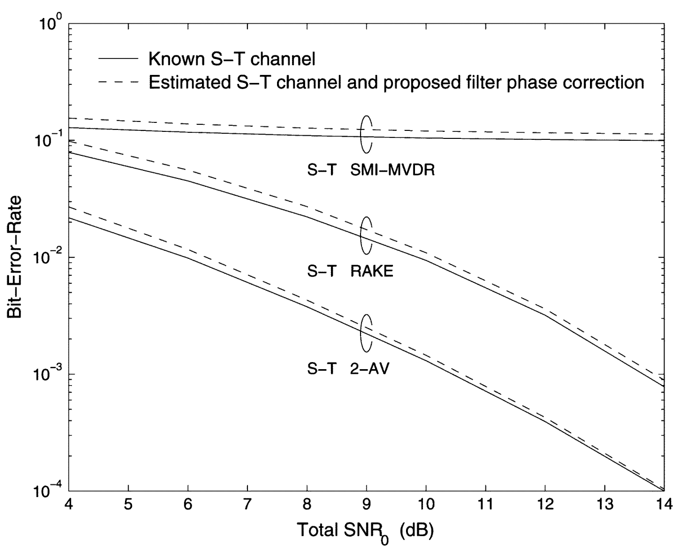

Fig. 5. BER versus total SNR of the user of interest. Sample support is $J = 256$ and includes $P = 5$ pilot bits.

VI. Conclusion

Blind channel-estimation procedures return phase-ambiguous channel impulse response estimates. Therefore, coherent receiver designs that rely on such channel estimates cannot be pursued. The phase-ambiguity problem can be by-passed by differential modulation and detection that leads to decision statistics that are independent of the channel phase. Of course, differential modulation schemes come at a certain, well-documented, performance loss in comparison with their direct modulation counterparts.

In this paper, we considered an alternative approach to differential modulation. We developed a supervised MS-optimum phase-recovery procedure and we showed that phase correction for any linear filter receiver takes the form of a simple projection operation. Conveniently, any known blind channel-estimation algorithm can be used to provide the initial phase-ambiguous estimate. Then, a small (single digit) record of pilot information symbols is sufficient for the supervised MS algorithm to recover effectively the unknown phase. Our presentation was given in the context of joint ST adaptive linear filter receivers for BPSK DS-CDMA antenna-array systems. The studies showed that supervised-phase-corrected ST adaptive receivers (in particular, AV type) based on blind subspace ST channel estimates lead to powerful adaptive MAI suppression. As a realistic numerical example, we used a data record size (packet size) of 256 bits that includes 5 pilot bits (about 2% pilot signaling).

Appendix

Proof of Proposition 3: To examine the rank of $\mathbf{M}_0(\vec{\theta})$, we recall the following two useful identities from linear algebra [42]:

$$\text{rank}\,(\mathbf{A}^H\mathbf{A}) = \text{rank}\,(\mathbf{A}) \tag{33}$$
$$\text{rank}\,(\mathbf{AB}) = \text{rank}\,(\mathbf{B}) - \dim(\text{null}\,(\mathbf{A}\bigcap\text{range}\,(\mathbf{B})). \tag{34}$$

For any parameter vector $\vec{\theta} \in \mathcal{R}^N$

$$\begin{aligned}\text{rank}\,(\mathbf{M}_0(\vec{\theta}))
&= \text{rank}\,\left(\mathbf{B}(\vec{\theta})^H\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0\mathbf{B}(\vec{\theta})\right)\\
&= \text{rank}\,\left(\mathbf{U}_n^H\mathbf{A}_0\mathbf{B}(\vec{\theta})\right)\\
&= \text{rank}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta})) - \dim\left(\text{null}\,\left(\mathbf{U}_n^H\right)\bigcap\text{range}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta}))\right).\end{aligned} \tag{35}$$

Applying one more time (34) to the product $\mathbf{A}_0\mathbf{B}(\vec{\theta})$, we obtain

$$\begin{aligned}\text{rank}\,(\mathbf{M}_0(\vec{\theta}))
&= \text{rank}\,(\mathbf{B}(\vec{\theta})) - \dim(\text{null}\,(\mathbf{A}_0)\bigcap\text{range}\,(\mathbf{B}(\vec{\theta})))\\
&\quad - \dim\left(\text{null}\,\left(\mathbf{U}_n^H\right)\bigcap\text{range}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta}))\right).\end{aligned} \tag{36}$$

Since $\text{rank}\,(\mathbf{A}_0^s) = N$ and $\mathbf{A}_0 = \mathbf{A}_0^s \odot \mathbf{I}_M$,

$$\text{rank}\,(\mathbf{A}_0) = MN. \tag{37}$$

$\mathbf{A}_0$ is an $[M(L-N+1)] \times MN$ matrix, hence [43]

$$\dim(\text{null}\,(\mathbf{A}_0)) + \text{rank}\,(\mathbf{A}_0) = MN. \tag{38}$$

From (37) and (38), we obtain $\dim(\text{null}\,(\mathbf{A}_0)) = 0$, therefore

$$\text{null}\,(\mathbf{A}_0) = \{\mathbf{0}\}. \tag{39}$$

It can also be shown that for any $\vec{\theta} \in \mathcal{R}^N$

$$\text{rank}\,(\mathbf{B}(\vec{\theta})) = N. \tag{40}$$

From (36), (39), and (40), we obtain

$$\begin{aligned}\text{rank}\,(\mathbf{M}_0(\vec{\theta})) &= N - \dim(\{\mathbf{0}\}) - \dim\left(\text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta}))\right)\\ &= N - \dim\left(\text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0\mathbf{B}(\vec{\theta}))\right). \end{aligned} \quad (41)$$

By hypothesis 2), we conclude that

$$\text{rank}\,(\mathbf{M}_0(\vec{\theta})) = \begin{cases} N-1, & \vec{\theta} = \vec{\theta}_0 \\ N, & \vec{\theta} \neq \vec{\theta}_0. \end{cases} \quad (42)$$

The matrix $\mathbf{M}_0(\vec{\theta})$ is of size $N \times N$. Therefore, (42) implies that there is a unique parameter vector $\vec{\theta} = \vec{\theta}_0$ that makes the matrix $\mathbf{M}_0(\vec{\theta})$ singular, and for this specific $\vec{\theta}_0$ choice, there is a unique (within a complex scalar ambiguity) "zero eigenvector" of $\mathbf{M}_0(\vec{\theta}_0)$. □

Proof of Proposition 5: We use properties (33) and (34) to examine the rank of $\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0$

$$\begin{aligned}\text{rank}\ &\left(\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0\right)\\ &= \text{rank}\ \left(\mathbf{U}_n^H\mathbf{A}_0\right)\\ &= \text{rank}\,(\mathbf{A}_0) - \dim\left(\text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0)\right). \end{aligned} \quad (43)$$

We recall that $\mathbf{A}_0$ is of size $M(L-N+1) \times MN$, hence

$$\text{rank}\,(\mathbf{A}_0) \leq MN. \quad (44)$$

We also note that the signal contribution $\mathbf{A}_0\mathbf{h}_0$ of user 0 belongs both to the signal subspace and to the subspace spanned by the columns of $\mathbf{A}_0$

$$\begin{aligned} &\left.\begin{array}{l}\mathbf{A}_0\mathbf{h}_0 \in \text{null}\ \left(\mathbf{U}_n^H\right)\\ \mathbf{A}_0\mathbf{h}_0 \in \text{range}\,(\mathbf{A}_0)\end{array}\right\}\\ &\quad\Rightarrow \mathbf{A}_0\mathbf{h}_0 \in \text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0)\\ &\quad\Rightarrow \text{range}\,(\mathbf{A}_0\mathbf{h}_0) \subseteq \text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0). \end{aligned} \quad (45)$$

Therefore

$$\dim\left(\text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0)\right) \geq 1 \quad (46)$$

with equality if and only if null $(\mathbf{U}_n^H)\bigcap$range $(\mathbf{A}_0)$ = range $(\mathbf{A}_0\mathbf{h}_0)$.

The matrix $\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0$ is of size $MN \times MN$. Therefore, the vector $\mathbf{h}_0$ is uniquely determined by *Proposition 4* if and only if rank $(\mathbf{A}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_0) = MN - 1$. By considering (43), (44), and (46), a necessary and sufficient condition for the latter equality to hold true is

$$\text{rank}\,(\mathbf{A}_0) = MN \quad (47)$$

$$\dim\left(\text{null}\ \left(\mathbf{U}_n^H\right)\bigcap \text{range}\,(\mathbf{A}_0)\right) = 1. \quad (48)$$

Since $\mathbf{A}_0 = \mathbf{A}_0^s \odot \mathbf{I}_M$, (47) is equivalent to rank $(\mathbf{A}_0^s) = N$. Finally, (48) holds true if and only if null $(\mathbf{U}_n^H)\bigcap$range $(\mathbf{A}_0)$ = range $(\mathbf{A}_0\mathbf{h}_0)$. □

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