Single-antenna Coherent Detection of Collided FM0 RFID Signals

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Abstract—This work derives and evaluates single-antenna detection schemes for collided radio frequency identification (RFID) signals, i.e. simultaneous transmission of two RFID tags, following FM0 (biphase-space) encoding. In sharp contrast to prior art, the proposed detection algorithms take explicitly into account the FM0 encoding characteristics, including its inherent memory. The detection algorithms are derived when error at either or only one out of two tags is considered. It is shown that careful design of one-bit-memory two-tag detection can improve bit-error-rate (BER) performance by 3dB, compared to its memoryless counterpart, on par with existing art for single-tag detection. Furthermore, this work calculates the total tag population inventory delay, i.e. how much time is saved when two-tag detection is utilized, as opposed to conventional, single-tag methods. It is found that two-tag detection could lead to significant inventory time reduction (in some cases on the order of 40%) for basic framed-Aloha access schemes. Analytic calculation of inventory time is confirmed by simulation. This work could augment detection software of existing commercial RFID readers, including single-antenna portable versions, without major modification of their RF front ends.

Index Terms—RFID, Gen2, FM0 coding, collision detection

I. INTRODUCTION

Significant progress has been made since the invention and first use of RFID, i.e. transmission of an identification bit string by means of signal reflection rather than active radiation [1]. Today, relevant applications have emerged in various domains, including logistics/inventory management [2], backscatter sensor networks [3]–[5], or even musical instruments [6], [7]. Anti-collision of RFIDs in the widely-used UHF industry standard EPC Class 1 Generation 2 (Gen2, also ISO-registered as 18000 – 6C) [8] is based on framed-Aloha, i.e. time is split in frames and each frame in slots; tags randomize their broadcast to minimize probability of simultaneous transmission of more than one tags at a given slot [9], [10]. In other words, tag collision is harmful only when the RFID reader cannot detect information from more than one simultaneous tag transmissions. However, Gen2 does not specify reader detection and leaves open the possibility to exploit simultaneous tag transmissions. It is remarked that older RFID standardization attempts considered binary tree splitting methods for collision-free tag access, which were later abandoned in Gen2.

The scientific community has recently attempted to redefine the notion of RFID collision, by proposing new receiver methods that could withstand simultaneous reception of more than one tags. Work in [11] is perhaps one of the first that utilized a custom, software-defined radio monitor for RFID signals and tested separation of non-Gen2 tags with DBPSK modulation. Work in [12] tested high signal-to-noise ratio (SNR) detection methods for simultaneous reception of more than one non-Gen2 tags and was based on meticulous observation of the in-phase (I) and quadrature (Q) components of the received backscattered signal, after transmission from more than one tags. Careful modeling of the backscatter radio channel and the received I and Q components were further exploited in [13] with zero-forcing techniques. Furthermore, throughput enhancement of framed Aloha was theoretically calculated. Multi-antenna detection, based on blind source separation of zero constant-modulus signals, was proposed in [14] and experimentally validated in [15].

However, the aforementioned techniques above were either based on multi-antenna techniques or (even at the case of single-reader antenna) did not exploit the characteristics of tag transmission encoding, including inherent memory for the special case of FM0. Also known as biphase-space, FM0 is one of the two encoding schemes used in Gen2 tags and is broadly utilized in commercial tags (the other scheme is Miller or biphase-mark encoding).

In this work, we explicitly take into account the FM0 encoding characteristics, including its inherent memory and derive and evaluate single-antenna detection schemes for simultaneous transmission of two tags. Our developments do not assume a specific channel (or I/Q) model and were inspired from work in [16], which presented BER-optimal detection of a single FM0-encoded RFID tag. We follow the same signal model which is validated by experimental measurements using a custom software-defined radio receiver (sniffer). Specifically, utilization of the magnitude of the in-phase/quadrature (I/Q) signal eliminates the frequency offset between RFID reader and sniffer. Furthermore, we focus on tag population inventory delay, i.e. we compute how much time is saved when two-tag detection is utilized as opposed to conventional single-tag detection. Inventory time is measured in slots and calculated reduction is performed through theoretic calculation and confirmed by simulation. In that way, the benefits of the proposed signal detection techniques are highlighted in the context of RFID inventory applications.

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Contributions of this work are summarized below:

A. Single-antenna methods that exploit FM0 encoding are derived for two-tag detection without any specific modeling assumptions regarding the backscatter channel or reader front end (I and Q components).

B. At the physical layer, it is shown how one-bit memory of FM0 encoding can be also exploited in two-tag detection to improve performance by 3 dB, compared to maximum-likelihood (ML) memoryless two-tag detection. Analytic BER results are confirmed by simulation.

C. At the medium access control (MAC) layer, analytic results are offered regarding tag population inventory delay reduction (as opposed to throughput) for a basic version of framed-Aloha. Analysis is confirmed by simulation.

The single-antenna detection methods of this work could be readily applied in multi-antenna commercial RFID readers (e.g. Gen2), especially those that operate in antenna switching mode, without any modification of their RF front end. Furthermore, this work could enhance performance in portable RFID readers, where physical size forbids more than one antennas (especially in UHF). The proposed methods accelerate the inventory of a given tag population and their performance is quantified at both physical and MAC layers.

Section II describes the basic assumptions and formulates the problem studied in this work. Section III studies a multitude of memoryless or memory-assisted single-antenna detection methods for simultaneous transmission of two FM0-encoded tags. Section IV analytically calculates the overall delay (in number of slots) for inventory of many tags as a function of conventional or nonconventional (the latter are proposed in this work) reader detection policies. Finally, Sections V and VI offer the simulation results and conclusion, respectively.

II. Problem Formulation and System Model

In FM0 encoding, signal (line) level always changes at the bit boundaries. Moreover, signal level changes at the middle of the bit period only for bit “0” (while for bit “1” the level is kept constant) as depicted in Fig. 1. Thus, encoding of a single FM0 bit requires memory of the previous bit so that signal levels are modified accordingly at the bit boundaries. Each FM0-encoded bit can be represented as a vector of two half-bit constants of the form \( [\pm a \quad \pm a]^T \) where sign of \( a \) depends on the transmitted bit as well as the signal memory (i.e. previous transmission level).

To validate the signal model of [16] that we follow in this work, we utilized a simple and low-cost measurement setup (Fig. 2-(c)) that consists of a commercial UHF Gen2 reader, two FM0 tags, and a USRP software-defined radio (SDR) with a broadband daughterboard tuned at 865 MHz; the SDR acts as a low-cost Gen2 monitor (sniffer). A SDR-based Gen2 monitor was also recently developed in [17]. With custom software developed throughout this work, conversation between two tags and the reader was recorded at the sniffer. The down-converted baseband signal magnitude \( \sqrt{T^2(t) + Q^2(t)} \) at the sniffer (where \( I(t) \) and \( Q(t) \) represent the in-phase and quadrature signal components, respectively) is depicted in Fig. 2-(a), where it is shown that on top of a DC constant there is encoded information (due to the carrier transmitted from the reader and scattered back from the tags).

The signal part depicted as “collision” is magnified and zero-centered in Fig. 2-(b), which depicts the measured downconverted sum of two FM0 signals; such “collision” corresponds to simultaneous transmission (through backscatter) during the query phase of the Gen2 protocol, when random 16-bit ID information is transmitted by each tag (a.k.a. RN16). The above measurement validates the signal model of [16] followed in this work; furthermore, processing of \( \sqrt{T^2(t) + Q^2(t)} \) eliminates the frequency offset between reader and sniffer. It is noted however that at an operating RFID reader, where the detection methods proposed in this work could be implemented, there is no frequency offset between the reader’s transmit and receive paths (i.e. the reader uses the same oscillator for up- and down-conversion) [18].

Given that tag transmission (via backscatter) in commercial RFID protocols (e.g. Gen2) is always initiated and directed by the reader, while the typical range of such systems is on the order of a few meters and the minimum bit duration is on the order of a few microseconds, one would expect the two collided tag signals to arrive at the sniffer (or the reader) with negligible time difference compared to the bit duration and aligned bit boundaries. Thus, detecting such collided information is simpler than prior art that addresses separation of co-channel signals with misaligned bits. For example, one could first ignore the weak signal, detect bits from the strongest signal, remodulate it and cancel it from the aggregate received waveform in the frequency domain and then perform detection of the weakest signal (e.g. see relevant work in [19] and references therein). In this work, the fact that tags respond to reader signals in a slotted fashion is explicitly taken into account. Furthermore, the bit alignment assumption is validated by experimental measurements and the followed formulation facilitates the exploitation of the inherent memory of the FM0 line encoding. On the other hand, the amplitudes of the received tag signals also depend on the particular phases of their backscattered carrier (as well as on range from reader) and, thus, should be in general different.

Indeed, the aforementioned assumptions above are confirmed by measurements. Fig. 2-c depicts how the measured signal looks from two collided FM0 tags. Similar measurement plots have also appeared in [15], [17], and [20]. One could observe four different amplitude levels stemming from the addition of the two tags. There are also interesting spikes either due to noise or due to bit duration mismatch; the latter is due to the fact that RFID tags do not typically have accurate crystals for timing purposes but instead derive clocking signals from the reader-transmitted carrier through low-cost passive
components with, in general, variable manufacturing tolerance [18].

Consequently, after pulse-matched filtering and sampling at the RFID reader, the in-phase (or quadrature)\(^1\) component of the collided signal during one bit period can be represented by a vector \([x_0 \ x_1]^T\) of two half-bit symbols, where each half-bit symbol belongs in \(S = \{ s_0 = -a - b, s_1 = -a + b, s_2 = a - b, s_3 = a + b \}\). Slow-fading can be assumed, i.e. \(a, b\) remain constant during reception given the limited number of considered bits, either in RN16 or in the actual tag ID (96 bits in electronic product code). We also assume coherent reception, i.e. the constants \(a, b\) are considered known at the receiver. Such knowledge can be acquired through estimation using specialized pilot signals or could be estimated by the observation of the four amplitude levels of the aggregate downconverted and filtered data. It is remarked that, if \(a = b\), then \(s_1 = s_2 \in S\) and information is lost, i.e. separation of tags A and B fails. In general, \(a \neq b\) and their power ratio will be explicitly taken into account. The power ratio of signals from two tags can easily vary by several dBs, even for equidistant tags from the reader, as experimentally measured in [21]. Tag chip mismatching and and chip variability (e.g. chips produced by different vendors) further increase the power variability of the received backscattered signals received at the reader. Without loss of generality, we assume \(a > b > 0\) throughout this work.

Under the above assumptions, the received signal can be written in vector form as:

\[
y \triangleq \begin{bmatrix} y_0 \\ y_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} + \mathbf{n},
\]

\(^1\)For simplicity of the derivations and clarity of the presentation, in this work we consider processing of the in-phase (or quadrature) component only. Our developments can be extended to joint processing of the in-phase and quadrature components in a straightforward manner.
where \([x_0 \ x_1]^T \in S^2\) is the collided information signal and \(n = [n_0 \ n_1]^T\) represents additive white Gaussian noise (AWGN) where \(n_0, n_1\) are independent, zero-mean Gaussian variables with variance \(\sigma^2\).

The minimum distance rule (ML) given measurement \(y_i, i \in \{0, 1\}\) and transmitted constellation \(S\), with decision boundaries depicted in Fig. 3, provides the following conditional error probability:

\[
\Pr(\hat{x}_i \neq x_i|y_i) = \Pr(\hat{x}_i \neq x_i|y_i)
\]

\[
= Q\left(\frac{b}{\sigma}\right), \quad i = 0, 1,
\]

\[
= Q\left(\frac{a-b}{\sigma}\right) + Q\left(\frac{a+b}{\sigma}\right), \quad i = 0, 1,
\]

where \(Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2}dt\) is the Q function. The expressions above will be found useful throughout the document.

The above modeling approach is sufficient for the examination of the proposed two-tag detection methods. For complex modeling of the backscatter radio channel, the interested reader could refer to several works, including [3], [13], and [22].

### III. Detection Techniques

In subsections III-A, III-B, and III-C, we derive three methods for detection of both tag A and tag B FM0 information, alongside their respective (single-bit and bit-pair) error probabilities. In subsections III-D and III-E, two methods are derived for single-tag detection.

#### A. Method I: Memoryless Detection based on ML and two half-bits

This method performs independent detection of the two half-bit symbols (according to decision areas of Fig. 3) and then, based on the findings, final decision on both tag A and B information is jointly made. The detection method is summarized below:

- Detect \(\hat{x}_0 \in S\) from \(y_0\), applying a ML (i.e. minimum-distance) rule.
- Detect \(\hat{x}_1 \in S\) from \(y_1\), applying a ML (i.e. minimum-distance) rule.
- Decide in favor of \(H_i\) (i.e. \(\hat{H} = H_i\)), \(i \in \{0, 1, 2, 3\}\), from sign change between \(\hat{x}_0\) and \(\hat{x}_1\). If sign of \(a\) in \(\hat{x}_0\) is different than in \(\hat{x}_1\), then \(\text{tag}_A = 0\), otherwise \(\text{tag}_A = 1\). Similarly, if sign of \(b\) in \(\hat{x}_0\) is different than in \(\hat{x}_1\), then \(\text{tag}_B = 0\), otherwise \(\text{tag}_B = 1\).

For example, if \(\hat{x}_0 = a - b = s_2\) and \(\hat{x}_1 = -a - b = s_0\), then the bit estimates for tags A and B are \(\text{tag}_A = 0\) and \(\text{tag}_B = 1\), respectively. Such a case corresponds to hypothesis \(H_2\) according to Table I-A. It is remarked that Method 1 does not require knowledge of the noise variance \(\sigma^2\) per half-bit, at the receiver.

It is straightforward to compute error (or, equivalently, zero-error) performance of the above detection method. Observing that, under hypothesis \(H_0\) and FM0 signaling, only transitions between \(s_0\) and \(s_3\) or between \(s_1\) and \(s_2\) are allowed, the following conditional error probability can be readily calculated:

\[
\Pr(\hat{H}_i, i \neq 0|H_0) = 1 - Pr(\hat{H}_0|H_0)
\]

\[
= 1 - \frac{1}{4}\left\{[1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_0)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_1)]
\right.

\[
+ [1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_1)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_2)]
\right.

\[
+ [1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_2)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_1)]
\]

\[
+ [1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_3)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_0)]
\right\}

\[
= 1 - \frac{1}{2}\left\{[1 - Q(0/\sigma)]^2 + [1 - Q(0/\sigma)]^2 - Q(0/\sigma)Q((a-b)/\sigma)\right\}.
\]

Under hypothesis \(H_1\) and FM0 signaling, transitions between \(s_0\) and \(s_1\) or between \(s_2\) and \(s_3\) are allowed. Thus, \(H_i, i \neq 1|H_1\):

\[
Pr(\hat{H}_i, i \neq 1|H_1) = 1 - Pr(\hat{H}_0|H_1)
\]

\[
= 1 - \frac{1}{4}\left\{[1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_0)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_1)]
\right.

\[
+ [1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_1)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_0)]
\right.

\[
+ [1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_2)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_3)]
\right.

\[
+ [1 - Pr(\hat{x}_0 \neq x_0|y_0 = s_3)][1 - Pr(\hat{x}_1 \neq x_1|y_0 = s_2)]
\right\}

\[
= 1 - \{[1 - Q(0/\sigma)]^2 - Q(0/\sigma)Q((a-b)/\sigma)\}.
\]

Under similar reasoning, it can be shown that:

\[
Pr(\hat{H}_i, i \neq 2|H_0) = Pr(\hat{H}_i, i \neq 1|H_1), \quad (6)
\]

\[
Pr(\hat{H}_i, i \neq 3|H_3) = Pr(\hat{H}_i, i \neq 0|H_0). \quad (7)
\]

Therefore, probability of detection error in at least one of the two tags is given by:

\[
Pr((\text{tag}_A, \text{tag}_B) \neq (\text{tag}_A, \text{tag}_B)) = \frac{1}{4}\sum_{j=0}^{3}Pr(H_i, i \neq j|H_j) =
\]

\[
= Q\left(\frac{b}{\sigma}\right)\left[2 - Q\left(\frac{b}{\sigma}\right) - Q\left(\frac{a-b}{\sigma}\right)\right]
\]

\[
+ Q\left(\frac{a-b}{\sigma}\right)\left[1 - \frac{1}{4}Q\left(\frac{a-b}{\sigma}\right)\right]. \quad (8)
\]

If we restrict the definition of detection error solely with respect to tag A, i.e. correct (or erroneous) detection of tag B is indifferent, and follow Method 1, then the error probability can be also readily calculated. Decision areas for half-bit detection in Fig. 3 become \((y_i < 0\) for \(\hat{x}_i = s_0\) or \(s_1\) and \(y_i > 0\) for \(\hat{x}_i = s_2\) or \(s_3\), \(i \in \{0, 1\}\)) and conditional error probabilities...
of eqs. (2) and (3) are modified to:

\[
\Pr( \hat{x}_i = s_2 \text{ or } s_3 | x_i = s_0 ) = \Pr( \hat{x}_i = s_0 \text{ or } s_1 | x_i = s_3 ) = Q\left( \frac{a + b}{\sigma} \right), \quad i = 0, 1, \\
\Pr( \hat{x}_i = s_2 \text{ or } s_3 | x_i = s_1 ) = \Pr( \hat{x}_i = s_0 \text{ or } s_1 | x_i = s_2 ) = Q\left( \frac{a - b}{\sigma} \right), \quad i = 0, 1.
\]  

(9)  

(10)

Following the same derivation of eqs. (4)-(7), the bit error probability of detection of tag A information with Method 1 becomes:

\[
\Pr(\hat{\text{tag}}_A \neq \text{tag}_A) = \left[ Q\left( \frac{a + b}{\sigma} \right) + Q\left( \frac{a - b}{\sigma} \right) \right] \\
\times \left\{ 1 - \frac{1}{4} \left[ Q\left( \frac{a + b}{\sigma} \right) + Q\left( \frac{a - b}{\sigma} \right) \right] \right\}.
\]

(11)

B. Method 2: ML Memoryless Detection

The previous method performs optimal hard decision per half-bit and then decides in favor of the detected hypothesis based on the half-bit hard decisions. In the following, we base our decision directly on the entire bit duration (without making half-bit decisions) and derive the ML detection rule. It is reminded that \( x_0 \) denotes the first half-bit symbol.

Under hypothesis \( H_0 \), both tags change their signal levels after the end of the first half-bit. Thus, signal \( a + b \) becomes \( -a - b \), signal \( a - b \) becomes \( -a + b \), and so forth. As a result, the conditional pdf of the received two-sample vector becomes:

\[
f(\mathbf{y}|H_0) = \frac{1}{4} \sum_{j=0}^{3} f(\mathbf{y}|H_0, x_0 = s_j) = \frac{1}{4}g(-a - b, a + b) + \frac{1}{4}g(a - b, -a + b) \\
+ \frac{1}{4}g(a + b, -a - b) + \frac{1}{4}g(-a + b, a - b) = k_2 e^{-2\hat{b}cosh\left\{ \frac{(a + b)(y_1 - y_0)}{\sigma^2} \right\}} \\
+ k_2 e^{2\hat{b}cosh\left\{ \frac{(a - b)(y_1 - y_0)}{\sigma^2} \right\}},
\]

(12)

where \( k_2 \) is a positive term and \( g(a_0, a_1) \triangleq N([a_0^T, a_1^T ]^{T}, [\sigma^2 I_{2 \times 2} ; \Sigma_{\mathbf{y}}]) \). The other three conditional pdfs are calculated similarly and equal to:

\[
f(\mathbf{y}|H_1) = k_2 \cosh\left\{ \frac{(a - b)y_0 - (a + b)y_1}{\sigma^2} \right\} \\
+ k_2 \cosh\left\{ \frac{(a + b)y_0 - (a - b)y_1}{\sigma^2} \right\},
\]

(13)

\[
f(\mathbf{y}|H_2) = k_2 \cosh\left\{ \frac{(a + b)y_0 + (a - b)y_1}{\sigma^2} \right\} \\
+ k_2 \cosh\left\{ \frac{(a - b)y_0 + (a + b)y_1}{\sigma^2} \right\},
\]

(14)

\[
f(\mathbf{y}|H_3) = k_2 e^{2\hat{b}cosh\left\{ \frac{(a - b)(y_0 + y_1)}{\sigma^2} \right\}} \\
+ k_2 e^{-2\hat{b}cosh\left\{ \frac{(a + b)(y_0 + y_1)}{\sigma^2} \right\}}.
\]

(15)

Notice that the above expressions require knowledge of \( \sigma^2 \). Thus, the ML detector is given by

\[
\hat{\mathbf{H}} = \arg \max_{\mathbf{H} \in \{ H_0, H_1, H_2, H_3 \}} \{ f(\mathbf{y}|\mathbf{H}) \}.
\]

(16)

Although, given knowledge of \( \sigma^2 \) at the receiver, Method 2 outperforms Method 1 in terms of BER by definition, the two detectors’ error probabilities practically coincide with each other, as will be demonstrated with results. Such observation holds when bit-pair error probability (i.e. both tags) as well as when single-bit error probability (i.e. tag A only) is of interest. Such result can be explained by the fact that the two half-bit observations of Method 1 constitute sufficient statistics for memoryless detection and hence performance in not degraded compared to Method 2. It is stressed however that Method 2 requires knowledge of the noise variance \( \sigma^2 \), while Method 1 does not.

C. Method 3: One-Bit-Memory-Assisted Detection

The previous two methods focus on the duration of a single bit (two consecutive half-bits) and, therefore, did not exploit the inherent memory of FM0 signaling. In Method 3, memory of FM0 signaling is exploited in detection of two collided FM0 signals by observing duration of exactly two bits: the bit under observation, half-bit before it, and half-bit after it. Similar mind-set was exploited by Simon and Divsalar [16] for detection of a single tag. They noticed that for ML single-bit (memoryless) detection there are four possible hypotheses to test; however, if half-bit before and half bit after are also observed, then there are only two hypotheses at the bit boundary (see shaded half-bits at Fig. 1). Below, we extend the idea in detection and separation of two FM0 tags.

With slight abuse of notation, we denote by \( y_0 \) the received half-bit signal before the bit boundary and \( y_1 \) the received half-bit signal after the bit boundary. Thus, there is a pair of measurements \((y_0, y_1)\) where \( y_1 \) corresponds to the first half-bit and \( y_0 \) corresponds to the second half-bit of the previous bit and a second pair of measurements \((y_0, y_1)\) where \( y_0 \) corresponds to the second half-bit and \( y_1 \) corresponds to the first half-bit of the next bit.

Given that the FM0 signal of each tag always changes levels at the bit boundaries, the possible transmitted symbols \( s_0, s_1, s_2, s_3 \) under either pair of measurements \((y_0, y_1)\), \( i = 0, 1 \), are depicted in Figures 4-a and 4-b. The detection algorithm works as follows:

- Detect \( \hat{x}_0 \in \mathcal{S} \) from \((y_0, y_1)\), applying a ML (i.e. minimum-distance) rule (Fig. 4-a).
- Detect \( \hat{x}_1 \in \mathcal{S} \) from \((y_0, y_1)\), applying a ML (i.e. minimum-distance) rule (Fig. 4-b).
- Decide in favor of \( H_i, i = 0, 1, 2, 3 \), based on \( \hat{x}_0, \hat{x}_1 \), according to Table I-B.

For example, if \( \hat{x}_0 = s_2 \) (Fig. 4-a) and \( \hat{x}_1 = s_0 \) (Fig. 4-b), then tag B level remains constant at \( -b \) (i.e. bit “1”) while
tag A level switches from +a to −a (i.e. bit “0”). Thus, we decide in favor of hypothesis H2, according to Table I-bottom. Similarly, the other entries above can be worked out.

The ML (i.e. minimum-distance) rule for \((y_0, y_1)^0\) or \((y_0, y_1)^1\) can be directly derived. Working on \((y_0, y_1)^0\) and \((y_0, y_1)^1\), the distances for the four transmitted symbols \(s_0, s_1, s_2, s_3\) are given by \(d_0^a, d_1^a, d_2^a\), and \(d_3^a\), \(i = 0, 1\), respectively, that are equal to:

\[
d_0^a[y_0, y_1] = d_3^a[y_0, y_1] = |y_0 - (a + b)|^2 + |y_1 - (-a - b)|^2, \tag{17}
\]

\[
d_1^a[y_0, y_1] = d_2^a[y_0, y_1] = |y_0 - (a - b)|^2 + |y_1 - (-a + b)|^2, \tag{18}
\]

\[
d_2^a[y_0, y_1] = d_1^a[y_0, y_1] = |y_0 - (-a + b)|^2 + |y_1 - (a - b)|^2, \tag{19}
\]

\[
d_3^a[y_0, y_1] = d_0^a[y_0, y_1] = |y_0 - (-a - b)|^2 + |y_1 - (a + b)|^2. \tag{20}
\]

Using \((y_0, y_1)^0\) and the distances of \(d_0^a, d_1^a, d_2^a\), and \(d_3^a\), in the following we describe how decision on \(\hat{x}_0\) is made. Similar approach is followed subsequently for the decision on \(\hat{x}_1\) (based on \((y_0, y_1)^1\) and \(d_0^1, d_1^1, d_2^1\), and \(d_3^1\)).

We detect \(\hat{x}_0 = s_0\) if and only if:

\[
d_0^a < d_1^a \Leftrightarrow y_0 - y_1 > 2a, \tag{21}
\]

\[
d_0^a < d_2^a \Leftrightarrow y_0 - y_1 > 2b, \tag{22}
\]

\[
d_0^a < d_3^a \Leftrightarrow y_0 - y_1 > 0. \tag{23}
\]

Having in mind that \(a > b\), we obtain:

\[
\hat{x}_0 = s_0 : \quad y_0 - y_1 > 2a. \tag{24}
\]

Working similarly for the other three hypotheses of Fig. 4-a, corresponding to the bit boundary with the previous bit, the ML decision areas become:

\[
\hat{x}_0 = \begin{cases} 
  s_0, & y_0 - y_1 > 2a, \\
  s_1, & 0 < y_0 - y_1 < 2a, \\
  s_2, & -2a < y_0 - y_1 < 0, \\
  s_3, & y_0 - y_1 < -2a.
\end{cases} \tag{25}
\]

The four decision areas above are depicted in Fig. 5.

Following similar steps for the hypotheses of Fig. 4-b, corresponding to the bit boundary with the next bit, we can derive the corresponding decision rules for \(\hat{x}_1\) (based on \((y_0, y_1)^1\) and \(d_0^1, d_1^1, d_2^1\), and \(d_3^1\)) which are simplified to:

\[
\hat{x}_1 = \begin{cases} 
  s_0, & y_0 - y_1 < -2a, \\
  s_1, & -2a < y_0 - y_1 < 0, \\
  s_2, & 0 < y_0 - y_1 < 2a, \\
  s_3, & y_0 - y_1 > 2a.
\end{cases} \tag{26}
\]

Erroneous detection of tag A or tag B FM0 signals occurs when detection from \((y_0, y_1)^0\) or detection from \((y_0, y_1)^1\) fails. The conditional error probabilities of such a detection scheme can be readily calculated. For example, the conditional error probability, given that \(x_0 = s_0\), equals:

\[
Pr(\hat{x}_0 \neq x_0|x_0 = s_0) = \int_{y_0=-\infty}^{\infty} \int_{y_1=-\infty}^{\infty} f(y_0, y_1|x_0 = s_0) \, dy_1 \, dy_0 \tag{27}
\]

\[
= \int_{y_0=-\infty}^{\infty} \int_{y_1=-\infty}^{\infty} g(a + b, -a - b) \, dy_1 \, dy_0. \tag{28}
\]

The above method requires numerical integration of the Q function. However, carefully observing that the method above improves the signal energy by exactly a factor of 2, since
duration of two-bits is exploited, as opposed to memoryless (single-bit) Method 1, it is inferred that the error performance of Method 3 improves over Method 1 with a SNR factor of two. Therefore, the probability \( \Pr((\hat{\text{tag}}_A, \hat{\text{tag}}_B) \neq (\text{tag}_A, \text{tag}_B)) \) that at least one of the two tag information is erroneously detected with Method 3 is given by:

\[
\Pr((\hat{\text{tag}}_A, \hat{\text{tag}}_B) \neq (\text{tag}_A, \text{tag}_B)) = Q\left(\sqrt{2}\frac{b}{\sigma}\right)\left[2 - Q\left(\sqrt{2}\frac{b}{\sigma}\right) - Q\left(\sqrt{2}\frac{a-b}{\sigma}\right)\right] \\
+ Q\left(\sqrt{2}\frac{a-b}{\sigma}\right)\left[1 - \frac{1}{4}Q\left(\sqrt{2}\frac{a-b}{\sigma}\right)\right]. \tag{29}
\]

Simulation results confirm the calculated expression above.

Furthermore, if detection of tag A information is important while tag B detected bits can be ignored, then performance of Method 3 can also be calculated. Following the same reasoning as above, BER performance \( \Pr(\hat{\text{tag}}_A \neq \text{tag}_A) \) of Method 3, when only tag A is of interest, is given by Eq. (11) with SNR improved by a factor of 2:

\[
\Pr(\hat{\text{tag}}_A \neq \text{tag}_A) = Q\left(\sqrt{2}\frac{a+b}{\sigma}\right) + Q\left(\sqrt{2}\frac{a-b}{\sigma}\right) \\
\times \left[1 - \frac{1}{4}\left(Q\left(\sqrt{2}\frac{a+b}{\sigma}\right) + Q\left(\sqrt{2}\frac{a-b}{\sigma}\right)\right)\right]. \tag{30}
\]

Numerical results confirm that the above expression coincides with simulation results. It is remarked that Method 3 does not require knowledge of the noise variance \( \sigma^2 \).

The previous Methods 1–3 targeted detection at both tags, even though performance was also calculated when only tag A was of interest. In the following subsections, ML detectors are derived when only tag A information is of interest (in the presence of tag B), with or without single-bit memory.

**D. Method 4: ML Memoryless Single-Tag Detection**

Working similarly as before, with \( x_0, x_1 \) the first and second half-bit and hypotheses in \( S \) of Fig. 3, the conditional pdfs are given by:

\[
f(\text{y}|\text{tag}_A = "0") = \frac{1}{8}\sum_{i=0}^{3} f(\text{y}|\text{tag}_A = "0", \text{tag}_B = "0", x_0 = s_i) \\
+ \frac{1}{8}\sum_{i=0}^{3} f(\text{y}|\text{tag}_A = "0", \text{tag}_B = "1", x_0 = s_i) \tag{31}
\]

\[
= k_4 \left( e^{-\frac{2a+b}{\sigma}} \sinh \left[ (a + b)(y_0 - y_1)/\sigma^2 \right] \\
+ e^{\frac{2a-b}{\sigma}} \sinh \left[ (a - b)(y_0 - y_1)/\sigma^2 \right] \\
+ \cosh \left[ (a(y_0 - y_1) + b(y_0 + y_1))/\sigma^2 \right] \\
+ \cosh \left[ (a(y_0 - y_1) - b(y_0 + y_1))/\sigma^2 \right] \right), \tag{32}
\]

and

\[
f(\text{y}|\text{tag}_A = "1") = \frac{1}{8}\sum_{i=0}^{3} f(\text{y}|\text{tag}_A = "1", \text{tag}_B = "0", x_0 = s_i) \\
+ \frac{1}{8}\sum_{i=0}^{3} f(\text{y}|\text{tag}_A = "1", \text{tag}_B = "1", x_0 = s_i) \tag{33}
\]

\[
= k_4 \left( e^{-\frac{2a+b}{\sigma}} \sinh \left[ (a + b)(y_0 + y_1)/\sigma^2 \right] \\
+ e^{\frac{2a-b}{\sigma}} \sinh \left[ (a - b)(y_0 + y_1)/\sigma^2 \right] \\
+ \cosh \left[ (a(y_0 + y_1) + b(y_0 - y_1))/\sigma^2 \right] \\
+ \cosh \left[ (a(y_0 + y_1) - b(y_0 - y_1))/\sigma^2 \right] \right), \tag{34}
\]

where \( k_4 \) is a positive term, common to both hypotheses. It is remarked that the above expressions require knowledge of \( \sigma^2 \) at the receiver.

The receiver simply decides \( \hat{\text{tag}}_A = "0" \) iff

\[
f(\text{y}|\text{tag}_A = "0") > f(\text{y}|\text{tag}_A = "1"),
\]

and \( \hat{\text{tag}}_A = "1" \) otherwise. Numerical results show that performance of such detector practically can coincide with performance of Method 1 (Eq. (11)).

**E. Method 5: One-Bit-Memory-Assisted Single-Tag Detection**

Finally, a single-bit memory-assisted detector is derived, when only tag A is of interest. Similarly to Method 3, we work separately on \( (y_0, y_1)^0 \) (corresponding to bit boundary with the previous bit) and \( (y_0, y_1)^1 \) (corresponding to bit boundary with the next bit) and decide in favor of hypotheses \( M^0 \) and \( M^1 \), respectively, where \( M^i, i = 0, 1 \), can be either \( M_0 \) (that corresponds to constellation signals \( s_0, s_1 \) of Fig. 4-a) or \( M_1 \) (that corresponds to constellation signals \( s_2, s_3 \) of Fig. 4-a).

Considering ML detection of \( M^0 \) from \( (y_0, y_1)^0 \), we utilize conditional pdfs:

\[
f((y_0, y_1)^0|M_0) = \frac{1}{2} f((y_0, y_1)^0|s_0) + \frac{1}{2} f((y_0, y_1)^0|s_1) \tag{35}
\]

\[
f((y_0, y_1)^0|M_1) = \frac{1}{2} f((y_0, y_1)^0|s_2) + \frac{1}{2} f((y_0, y_1)^0|s_3) \tag{36}
\]

and decide in favor of hypothesis \( M_0 \) , i.e. \( M^0 = M_0 \) iff:

\[
f((y_0, y_1)|M_0) > f((y_0, y_1)|M_1) \iff e^{-\frac{2a+b}{\sigma}} \sinh \left[ (a + b)(y_0 - y_1)/\sigma^2 \right] \\
+ e^{\frac{2a-b}{\sigma}} \sinh \left[ (a - b)(y_0 - y_1)/\sigma^2 \right] > 0. \tag{37}
\]

Thus, the receiver decides whether \( M^0 \) is \( M_0 \) or \( M_1 \) based on a pair of measurements \( (y_0, y_1)^0 \), where \( y_1 \) corresponds to the first half-bit and \( y_0 \) corresponds to the second half-bit of the previous bit. Similarly, the receiver decides whether \( M^1 \) is \( M_0 \) or \( M_1 \) based on a pair of measurements \( (y_0, y_1)^1 \) and Eq. (37), where \( y_0 \) corresponds to the second half-bit and \( y_1 \) corresponds to the first half-bit of the next bit. Finally, decision on tag A bit is made according to the following rule: if
\( M^0 = M^1 \) (i.e. both are M0 or both are M1), then \( \hat{\text{tag}}_A = "0" \), otherwise \( \hat{\text{tag}}_A = "1" \).

It is again remarked that the above expressions require knowledge of \( \sigma^2 \) at the receiver. Simulation results show that performance of the above detector practically coincides with performance of Method 3 (Eq. (30)).

IV. INVENTORY TIME BENEFITS

In this section, the impact of the above algorithms on the reduction of total inventory time (i.e. delay) for \( N \) tags is addressed, in the context of framed Aloha. The latter as already mentioned forms the basis of commercial RFID protocols (e.g. Gen2). High SNR analysis follows, assuming that when exactly one or two tags transmit in a given slot, their information can be correctly received. This section offers exact, closed-form formulas that compute the average inventory time and analysis results are validated by simulations.

In the basic version of framed Aloha, access is operated in frames where each frame is divided in \( L \) slots and tags at the beginning of each frame select independently and randomly one of the \( L \) slots to transmit their information. The beginning of each slot is marked by transmission of appropriate messages from a central controller. At the end of the frame, the central controller (e.g. reader in the context of RFID applications) estimates the number of remaining tags and advertises a new number \( L \) of total slots for the next frame. The remaining tags select independently and randomly the slot they are going to transmit in the next frame and the process continues until a predetermined number of tags is accessed. It is remarked that for the particular case of Gen2 the number of slots per frame is set at \( L = 2^Q \) and reader advertises \( Q \) at the beginning of each frame.

For a given number \( N \) of tag population and a number \( L \) of slots at a given frame, the probability of \( q \) tags transmitting at a given slot is described by the binomial term:

\[
\Pr(q)_{N,L} = \binom{N}{q} \left( \frac{1}{L} \right)^q \left( 1 - \frac{1}{L} \right)^{N-q}.
\]

Thus, successful transmission of tag information at a given slot can be readily calculated, also offering a measure of throughput.

First, it is assumed that tag collision occurs when more than one tags select the same slot, i.e. conventional processing at the reader. In that case, successful tag transmission occurs iff exactly one tag transmits at a slot and throughput per slot \( \rho_1 \), assuming detection at high SNR is given by:

\[
\rho_1 (N, L) \triangleq \Pr(\text{slot success}) = \Pr(q = 1)_{N,L} = N \left( \frac{1}{L} \right) \left( 1 - \frac{1}{L} \right)^{N-1}.
\]

Maximizing throughput per slot for a given number of slots \( L \) per frame offers the appropriate number of slots which, for the case of conventional reader processing, is equal to the number of tags:

\[
\max_L \{ \rho_1 (N, L) \} \Rightarrow \hat{\text{L}}_1(N) = N.
\]

Second, for nonconventional reader processing, e.g. when exactly one out of two tags can be decoded at the event of simultaneous transmission of two tags (as described in Section III), throughput per slot \( \rho_2 \), assuming detection at high SNR is given by:

\[
\rho_2 (N, L) = \Pr(q = 1)_{N,L} + \Pr(q = 2)_{N,L} = \frac{N}{L} \left( 1 - \frac{1}{L} \right)^{N-1} + \left( \frac{N}{2} \right)^2 \left( 1 - \frac{1}{L} \right)^{N-2}.
\]

Notice that, if we assumed that both tags (and not just one out of two) could be decoded at the case of simultaneous transmission of exactly two tags, then a factor of 2 would multiply the second probability term above. Maximization of the above throughput quantity offers the appropriate choice for number of slots per frame:

\[
\max_L \{ \rho_2 (N, L) \} \Rightarrow \hat{\text{L}}_2(N) = 1 + \sqrt{1 + \frac{N(N-3)}{2}}.
\]

Notice that, for \( N < 3 \) (i.e. \( N = 1 \) or \( N = 2 \)), the appropriate number of slots \( \hat{\text{L}}_2(N) = 1 \), as expected.

The basic framed Aloha control algorithm works as follows: maximize slot throughput per frame, i.e. set \( L(N) = \hat{\text{L}}_j(N) \), depending on how tag collision is defined (whether aforementioned detection algorithms of Section III are applied, in which case \( j = 2 \), or not, and thus \( j = 1 \)). When frame is completed (i.e. all slots are tested), update number \( N \) of backlogged tags (remaining number of tags to be read) and start a new frame.

It is remarked that the above algorithm assumes that the central controller (e.g. reader) has acquired an accurate estimate of the total number of tags \( N \). Such information can be inferred from the number of empty or collided slots and there are specific proposals in the literature, based on deterministic [9], probabilistic [23, 24], or recursive [25] techniques. More importantly, the above policy maximizes throughput per frame and not total number of frames (overall delay). It was recently shown that it could be beneficial to stop a frame before the total number of slots is tested (especially when probability of tag transmitting at remaining slots is small) and start a new frame with an updated slot number [26, 27]. Optimizing the framed Aloha policies are beyond the scope of this work.

The expected total number of frames \( F \) and expected total number of slots, required for the aforementioned basic framed Aloha scheme, can be readily calculated with the recursive equations (43)-(45) below, with initial condition \( N(1) = N \), where \( N \) denotes the total number of tags to be inventoried, index \( i \) denotes the frame number and index \( j \) indicates whether the reader can detect one tag information out of two collided signals (\( j = 2 \) or not (\( j = 1 \)):

\[
L(i) = \hat{\text{L}}_j (N(i)) ,\quad N(i+1) = N(i) - L(i) \rho_j (N(i), L(i)) ,\quad \sum_{i=1}^{F} L(i) \rho_j (N(i), L(i)) \geq a_p N.
\]

Eq. (43) sets the number of slots per frame according to Eq. (40) or Eq. (42), depending on the reader detection
method. Eq. (44) computes the expected number of remaining tags at the end of the frame, which is used to calculate the number of slots for the next frame. Eq. (45) sums all accessed tags and terminates the recursion if their sum is above the percentage \( a_p \) of the total tags that need to be read.

With the above recursion, the expected total number of frames \( F \) and slots per frame \( L(i) \) are estimated, when Eq. (40) or Eq. (42) are utilized, according to the basic framed Aloha scheme described above. Simulation results at Section V confirm the recursive theoretical calculation above. In either cases, the expected total number of slots required to access \( (a_p \times N) \) tags (e.g. \( a_p = 100\% = 1 \)) is given by:

\[
\sum_{i=1}^{F} L(i). \tag{46}
\]

With the above recursive methodology, inventory time benefits (i.e. delay reduction) can be readily calculated when detection techniques for two collided tags are utilized, as opposed to conventional detection (where collided signals of two tags are discarded). Additional analysis regarding variants of framed Aloha (e.g. Gen2) can be found in [15] and [28]. Finally, it is noted that the above methodology can be easily extended to cover the case of three (or more than three) tags transmitting at the same slot and the reader being able to detect the strongest. However, the probability of three tags selecting the same slot in framed Aloha systems is in general smaller than the probability of two tags transmitting at the same slot and thus, the observed benefits are not expected to be substantially better than the two-tag case [15].

V. NUMERICAL RESULTS

In the numerical results of this section, the signal-to-noise ratio (SNR) \( E_b/N_0 = b^2/\sigma^2 \), as well as the power ratio between the two baseband tag signals \( \Psi = a^2/b^2 \) are considered.

In Fig. 6, the BER as a function of SNR is depicted, when detection error at either tag (A or B) is considered. The power ratio between the two tags is set to \( \Psi = 6 \) dB (i.e. \( a = 2b \)) and Methods 1-3 (Subsections III-A-III-C) are tested (in Method 2, knowledge of noise variance \( \sigma^2 \) at the receiver is assumed). It is found that simulation matches analytical results of Method 1 (Eq. (8)), while Method 1 performs as well as Method 2. Such result could cause small surprise, given that Method 1 does not require any type of noise variance estimation. However, as already mentioned, Method 1 performs memoryless ML detection on half-bits with observations that offer sufficient statistics and thus, its performance should not differ from Method 2 (which is also ML, memoryless detection). It is noted however that Method 2 under imprecise knowledge of \( \sigma^2 \) offers deteriorated performance. Furthermore, simulation matches analysis results (Eq. (29)) for Method 3, which performs 3dB better than Method 1 due to intelligent exploitation of FM0 memory, as explained in Subsection III-C.

In Fig. 7, the previous experiments are repeated for Methods 1 and 3, with fixed SNR and variable \( \Psi \). As \( \Psi \) increases, the overall BER reaches a plateau. That is due to the fact that error at either tag is considered and, thus, the depicted BER is limited by the weakest tag (B in our case); by increasing \( \Psi \), errors at the strongest tag (tag A) are decreased but errors at the weakest tag are left unaffected. Thus, in cases where there is collision with a “weak” tag, the reader should only focus on the stronger tag.

Such strategy is examined in Fig. 8, where error only at tag A is considered and Methods 1-5 are tested for fixed \( \Psi \) and variable SNR. It can be seen that simulation matches analysis results for Method 1 (Eq. (11)), while Methods 2 and 4 perform no better than Method 1. Methods 2, 4, and 5 are assumed with perfect knowledge of noise variance \( \sigma^2 \). Fig. 8 shows that one could use Method 1 for single tag detection, when two tags collide, without any need for noise variance estimation and without performance loss, compared to the ML Method 4. A 3dB improvement can be further observed if Method 3 is utilized. Simulation results match analysis (Eq. (30)) for Method 3 which performs no worse than Method 5, even though the latter requires estimation of the noise variance \( \sigma^2 \) (assumed perfect in the depicted results).

Thus, Method 3 for single tag information extraction out of two collided tags, offers a simple and effective scheme without requiring noise variance estimates, by simple exploitation of FM0 memory. Fig. 9 repeats the aforementioned experiments for Methods 1 and 3 with variable \( \Psi \) and fixed SNR. It can be seen that Method 3 drops the BER to values on the order of
can be seen that reader's ability to detect and extract information for one out of two collided tag signals can significantly reduce overall inventory time (i.e., total number of slots) by 40% for SNR close to 10 dB and $\Psi = 6$ dB. One immediate question emerges: could additional FM0 memory (more than one bit) further reduce BER? The answer is negative and was already given by Simon and Divsalar for single-tag detection [16].

Finally, in Fig. 10 the expected total number of slots required to access $N$ tags is depicted, with the basic framed Aloha scheme of Section IV. Simulation matches the analytical results of Eq. (46) through the recursive methodology in Eqs. (43)-(45) for the whole population of tags (i.e., $A_0 = 1$). It can be seen that reader's ability to detect and extract information for one out of two collided tag signals can significantly reduce overall inventory time (i.e., total number of slots) by 40% (and even more for higher tag population $N$), depending on the total number of tags. Additional results relevant to inventory time reduction in a basic version of Gen2 (which is also a version of framed Aloha) can be found in [28].

VI. CONCLUSION

Commercial RFID protocols based on framed Aloha, including Gen2, can substantially benefit from the methodology of this work. What is needed is simple augmentation of detection algorithms at the reader, alongside the lines of this work. Single-bit memory-assisted algorithms are the basis of two-tag detection that could lead to inventory time reduction of $N$ tags on the order of 40% under certain conditions (e.g., high-SNR, sufficient tag signal separation $\Psi$) for basic framed Aloha access schemes without modification of reader RF front end. The algorithms could be of importance to single-antenna (e.g., portable) readers, as well as multiple-antenna readers (in antenna-switching mode).

REFERENCES


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