TABLE I
VALUES OF $\phi_{\max}$ CORRESPONDING
DAUBECHIES SCALING FUNCTION THROUGH $N = 2 \sim 5$

N	2	3	4	5
$\phi_{\max}$	1.73205080756590	1.76320523091274	1.09116496568688	1.00048422469579

TABLE II
COMPARISON OF THE APPROXIMATION ERRORS OF THE ORTHOGONAL PROJECTION

N	$E_O^{(2)}(f_1)$	$\hat{E}_O^{(2)}(f_1)$	$E_O^{(0)}(f_2)$	$\hat{E}_O^{(0)}(f_2)$
2	0.0303941162039	0.03561386723603	0.02058189833741	0.02791730842698
3	0.0087900005079	0.01468691131114	0.00945146526398	0.01802061589772
4	0.0027705547550	0.00397135019388	0.00516599819353	0.01343595552278
5	0.000911412144300	0.00140859211097	0.00315028315124	0.01112484670610

TABLE III
COMPARISON OF THE SYSTEMATIC ERRORS

N	$E_S^{(2)}(f_1)$	$\hat{E}_S^{(2)}(f_1)$	$E_S^{(0)}(f_2)$	$\hat{E}_S^{(0)}(f_2)$
2	0.06714880126461	0.07963502808127	0.04488877472676	0.06242499939145
3	0.01725001412647	0.02306191961446	0.01780321791304	0.03602837963144
4	0.00317231037715	0.00461325466474	0.00572367587279	0.01560766023259
5	0.000306524353272	0.00145402415052	0.00204837183965	0.01148366205909

TABLE IV
COMPARISON OF THE APPROXIMATION ERRORS OF THE PREFILTERED PROJECTION

N	$E_\phi^{(2)}(f_1)$	$\hat{E}_\phi^{(2)}(f_1)$	$E_\phi^{(0)}(f_2)$	$\hat{E}_\phi^{(0)}(f_2)$
2	0.07370823122739	0.08723580249536	0.04938235145921	0.06838316063790
3	0.01936045186155	0.02578584978889	0.02015650673344	0.04028382722880
4	0.004211831761173	0.00608717840745	0.00771025309781	0.02059427004645
5	0.000961576453500	0.00202443028168	0.00375767363225	0.01598864311071

V. CONCLUSIONS

In this correspondence, we analyze the approximation performance of a special FIR prefilter. The results show that the convergent rate of the prefiltered projection is the same as that of the orthogonal projection. In addition, for bandlimited signals, the quantitative estimates of the upper bounds of the three types of errors are obtained. Particularly for the Daubechies' orthogonal wavelet base, the estimated constant is optimal. Using these results, we can perform an initialization for the DWT.

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On the Robustness of the Linear Prediction Method for Blind Channel Identification with Respect to Effective Channel Undermodeling/Overmodeling

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***Abstract*—We study the performance of the linear prediction (LP) method for blind channel identification when the true channel is of order M, whereas the channel model is of order m, with $m < M$. By partitioning the true channel into the mth-order significant part and the unmodeled tails, we show that the LP method furnishes an approximation to the mth-order significant part. The closeness depends on the diversity of the mth-order significant part and the size of the unmodeled tails. Furthermore, we show that two frequently encountered claims concerning the LP method, namely, that a) the method is robust with respect to channel overmodeling and b) the performance of the method depends critically on the size of the first impulse response term, are not correct in realistic scenarios.**

***Index Terms*—Communications, multichannel system identification.**

I. INTRODUCTION

Many methods can claim exact SIMO channel identification, in the noiseless case, under the so-called length and zero conditions [1]–[4]. However, their behavior may change dramatically under more realistic conditions, including the presence of tails of "small" leading and/or trailing impulse response terms [5]. Robustness issues of blind channel identification methods in such scenarios are very important from a practical point of view [6] but are less well understood. We study the mean asymptotic performance offered by the linear prediction (LP) method when the true channel is of order M, whereas the channel model is of order m, with $m < M$. We term this case the mth-order LP method. For ease of presentation, we adopt a single-input/two-output channel setting. Extension to the case of p output channels with $p > 2$ is straighforward.

In Section II, we review the LP method for the exact order, exact statistics, noiseless case. In Section III, we bound the distance between the impulse response estimate furnished by the mth-order LP method and the mth-order significant part of the true channel, which will be defined in Section III. This distance depends on the diversity of the

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mth-order significant part, as measured by the smallest nonzero singular value of a certain filtering matrix, and the size of the unmodeled tails. In the course of our analysis, we show that two frequent claims concerning the LP method, namely, that a) the method is robust with respect to channel overmodeling and b) the performance of the method depends critically on the size of the first impulse response term, are not correct in realistic scenarios. In Section IV, we present simulations, and some concluding remarks are summarized in Section V.

II. LP Method: Exact Order Case

In this section and in order to fix notation, we review the basic steps of the LP method for blind channel identification, under the one-input/two-output noiseless channel setting. If the true order of the subchannels is M, then the output of the jth subchannel $x_i^{(j)}$, for $j = 1, 2$, is given by

$$x_i^{(j)} = \sum_{k=0}^{M} h_k^{(j)} s_{i-k}$$

where $h_k^{(j)}$ is the impulse response of the jth subchannel, and s_i is the input sequence, which is assumed to be zero-mean unit-variance i.i.d. The data vector $\mathbf{x}_L(i) \triangleq [\mathbf{x}_i^T \ \cdots \ \mathbf{x}_{i-L}^T]^T$, where superscript ^{T} denotes transpose, with $\mathbf{x}_i \triangleq [x_i^{(1)} \ x_i^{(2)}]^T$, can be expressed as $\mathbf{x}_L(i) = \mathcal{T}_L(\mathbf{h}_M)\mathbf{s}_{L+M}(i)$, where $\mathbf{h}_M \triangleq [\mathbf{h}_{(0)}^T \ \cdots \ \mathbf{h}_{(M)}^T]^T$ denotes the impulse response vector, with $\mathbf{h}_{(k)} \triangleq [h_k^{(1)} \ h_k^{(2)}]^T$, $\mathcal{T}_L(\mathbf{h}_M)$ is the $2(L+1) \times (L+M+1)$ generalized Sylvester matrix

$$\mathcal{T}_L(\mathbf{h}_M) \triangleq \begin{bmatrix} \mathbf{h}_{(0)} & \cdots & \mathbf{h}_{(M)} & & \\ & \ddots & & \ddots & \\ & & \mathbf{h}_{(0)} & \cdots & \mathbf{h}_{(M)} \end{bmatrix}$$

and $\mathbf{s}_{L+M}(i) \triangleq [s_i \ \cdots \ s_{i-L-M}]^T$. It is well known that if the subchannels of $\mathbf{h}_M$ do not share common zeros and $L \geq M-1$, then $\mathcal{T}_L(\mathbf{H}_M)$ has full-column rank.

In the sequel, we review how the Mth-order linear prediction error filter associated with $\mathbf{x}_i$ can be used for the identification of the Mth-order impulse response $\mathbf{h}_M$ [3]. At first, we compute the coefficients of the $2 \times 2M$ minimum mean-square error multichannel linear predictor

$$\underbrace{[\mathbf{A}_1 \ \cdots \ \mathbf{A}_M]}_{\mathcal{A}_M} = -\underbrace{[\mathbf{r}_1 \ \cdots \ \mathbf{r}_M]}_{\underline{\mathbf{r}}_M}\mathbf{R}_{M-1}^{-1}$$

where

$$\mathbf{R}_{M-1} \triangleq E\{\mathbf{x}_{M-1}(i)\mathbf{x}_{M-1}^T(i)\} = \mathcal{T}_{M-1}(\mathbf{h}_M)\mathcal{T}_{M-1}^T(\mathbf{h}_M)$$
$$\mathbf{r}_k \triangleq E\left\{\mathbf{x}_i\mathbf{x}_{i-k}^T\right\}.$$

If we define $\mathbf{D} \triangleq \mathbf{r}_0 + \mathcal{A}_M\underline{\mathbf{r}}_M^T$, then it can be shown that $\mathbf{D} = \mathbf{h}_{(0)}\mathbf{h}_{(0)}^T$. If λ and $\mathbf{v}$ are, respectively, the nonzero eigenvalue of the rank-one matrix $\mathbf{D}$, and its associated unit two-norm eigenvector, i.e., $\lambda = \|\mathbf{h}_{(0)}\|_2^2$ and $\mathbf{v} = \mathbf{h}_{(0)}/\|\mathbf{h}_{(0)}\|_2$, then an Mth-order zero-forcing zero-delay equalizer is given by the row vector $\mathbf{g}_M = (1/\sqrt{\lambda})\mathbf{v}^T[\mathbf{I} \ \mathcal{A}_M]$. The impulse response $\mathbf{h}_M$ can be identified via

$$\mathbf{h}_M = \mathbf{S}_M\mathbf{g}_M^T, \quad \text{with } \mathbf{S}_M \triangleq \begin{bmatrix} \mathbf{r}_0 & \cdots & \mathbf{r}_M \\ \vdots & \iddots & \\ \mathbf{r}_M & & \end{bmatrix}.$$

In [7, Eq. (3.10)], it is shown that $\mathbf{S}_M = \mathcal{H}_M(\mathbf{h}_M)\,\mathcal{T}_M^T(\mathbf{h}_M)$, with the $2\,(L+1) \times (L+M+1)$ matrix $\mathcal{H}_L(\mathbf{h}_M)$ defined, for $L \leq M$, as

$$\mathcal{H}_L(\mathbf{h}_M) \triangleq \begin{bmatrix} \mathbf{h}_{(0)} & \cdots & \mathbf{h}_{(L)} & \cdots & \mathbf{h}_{(M)} & \\ \vdots & \iddots & & \iddots & & \mathbf{O} \\ \mathbf{h}_{(L)} & \cdots & \mathbf{h}_{(M)} & & & \end{bmatrix}.$$

In [3], it was shown that the LP method is able to identify the unknown channel in the overmodeled, with respect to identically zero impulse response terms, exact statistics case. Furthermore, it was claimed that the algorithm is expected to perform well in the estimated statistics, noisy, overmodeled cases, although this was not supported by theoretical results.

In [3] and [4], it was claimed that if the first impulse response term $\mathbf{h}_{(0)}$ is "small," then the algorithm is expected to perform poorly. In [6] and [7], it has been argued that microwave radio channel impulse responses usually possess "small" leading and/or trailing terms. This comes from the fact that the impulse response $\mathbf{h}_M$ models both the shaping filters and the propagation through the channel. Does this mean that the LP method generically performs poorly in realistic cases? More generally, which are the factors that determine the robustness of the LP method in undermodeled/overmodeled cases? These are the questions we address in the sequel.

III. mth-Order LP Method

In order to study the mth-order LP method, we partition the true channel $\mathbf{h}_M$ into [5] [8]:

1) the *m-th order significant part*, which is the mth-order contiguous part of the true channel that has the largest energy among all its mth-order contiguous parts; it usually lies near the middle of the impulse response;
2) the *unmodeled tails*, which is the complementary part to the mth-order significant part; the unmodeled tails usually contain "small" leading and/or trailing terms.

Notationally, we express this partitioning for $0 \leq m_1 < m_2 \triangleq m_1 + m \leq M$, as

$$\mathbf{h}_M = \mathbf{h}_{m_1,m_2}^z + \mathbf{d}_{m_1,m_2}^z$$

where superscript ^{z} denotes "appropriately zero-padded," and $\mathbf{h}_{m_1,m_2}^z$ and $\mathbf{d}_{m_1,m_2}^z$ are defined by the expression at the bottom of the page. With $\mathbf{h}_{m_1,m_2}$, we denote the truncated mth-order significant part $\mathbf{h}_{m_1,m_2} \triangleq [\mathbf{h}_{(m_1)}^T \ \cdots \ \mathbf{h}_{(m_2)}^T]^T$. Our study proceeds in two steps. We first consider the mth-order LP method, assuming that the true channel is the mth-order significant part $\mathbf{h}_{m_1,m_2}$, and then, we study the behavior of the mth-order LP method upon augmenting this significant part with the tails.

$$\mathbf{h}_{m_1,m_2}^z \triangleq \Big[\underbrace{\mathbf{0}^T \ \cdots \ \mathbf{0}^T}_{m_1}\underbrace{\mathbf{h}_{(m_1)}^T \ \cdots \ \mathbf{h}_{(m_2)}^T}_{m+1}\underbrace{\mathbf{0}^T \ \cdots \ \mathbf{0}^T}_{M-m_2}\Big]^T, \mathbf{d}_{m_1,m_2}^z \triangleq \Big[\underbrace{\mathbf{h}_{(0)}^T \ \cdots \ \mathbf{h}_{(m_1-1)}^T}_{m_1}\underbrace{\mathbf{0}^T \ \cdots \ \mathbf{0}^T}_{m+1}\underbrace{\mathbf{h}_{(m_2+1)}^T \ \cdots \ \mathbf{h}_{(M)}^T}_{M-m_2}\Big]^T.$$

Thus, let us initially assume that the true impulse response is $\mathbf{h}_{m_1,m_2}$. Then, the mth-order autocorrelation matrix is

$$\mathbf{R}_m = \begin{bmatrix} \mathbf{r}_0 & \underline{\mathbf{r}}_m \\ \underline{\mathbf{r}}_m^T & \mathbf{R}_{m-1} \end{bmatrix} = \mathcal{T}_m(\mathbf{h}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}). \quad (1)$$

If the subchannels of $\mathbf{h}_{m_1,m_2}$ do not share common zeros, the mth-order LP method furnishes

$$\mathcal{A}_m = -\underline{\mathbf{r}}_m \mathbf{R}_{m-1}^{-1}, \quad \mathbf{D} = \mathbf{r}_0 + \mathcal{A}_m \underline{\mathbf{r}}_m^T = \mathbf{h}_{(m_1)}\mathbf{h}_{(m_1)}^T \quad (2)$$

$$\lambda = \left\|\mathbf{h}_{(m_1)}\right\|_2^2, \quad \mathbf{v} = \frac{\mathbf{h}_{(m_1)}}{\|\mathbf{h}_{(m_1)}\|_2}$$

$$\mathbf{g}_m = \frac{1}{\sqrt{\lambda}}\mathbf{v}^T[\mathbf{I}\ \mathcal{A}_m] \quad (3)$$

$$\mathbf{h}_{m_1,m_2} = \mathbf{S}_m\, \mathbf{g}_m^T \quad \text{with } \mathbf{S}_m = \mathcal{H}_m(\mathbf{h}_{m_1,m_2})\, \mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}). \quad (4)$$

Now, let us assume that the true impulse response is $\mathbf{h}_{m_1,m_2}^z$. Since

$$\mathcal{T}_m(\mathbf{h}_{m_1,m_2}^z)\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}^z) = \mathcal{T}_m(\mathbf{h}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2})$$

it is simple to show that by following the same sequence of steps, we can "identify" the nonzero part of $\mathbf{h}_{m_1,m_2}^z$, namely, $\mathbf{h}_{m_1,m_2}$.

Finally, let us consider our augmented problem, in which the true impulse response is $\mathbf{h}_M$, which, without loss of generality, is considered to be normalized to unit 2-norm, i.e., $\|\mathbf{h}_M\|_2 = 1$, under the assumption that $\mathbf{d}_{m_1,m_2}^z$ is small, i.e.,

$$\left\|\mathbf{d}_{m_1,m_2}^z\right\|_2 = \epsilon_m \ll 1. \quad (5)$$

In this case, the mth-order autocorrelation matrix is the perturbed version of $\mathbf{R}_m$, as shown in (6) at the bottom of the page. The linear predictor and the associated prediction error power are given by

$$\tilde{\mathcal{A}}_m = -\tilde{\underline{\mathbf{r}}}_m\tilde{\mathbf{R}}_{m-1}^{-1}, \quad \tilde{\mathbf{D}} = \tilde{\mathbf{r}}_0 + \tilde{\mathcal{A}}_m\tilde{\underline{\mathbf{r}}}_m^T. \quad (7)$$

Terms $\tilde{\lambda}$ and $\tilde{\mathbf{v}}$ are, respectively, the largest eigenvalue of $\tilde{\mathbf{D}}$ and its associated eigenvector, and

$$\tilde{\mathbf{g}}_m = \frac{1}{\sqrt{\tilde{\lambda}}}\tilde{\mathbf{v}}^T[\mathbf{I}\ \tilde{\mathcal{A}}_m]. \quad (8)$$

Finally, the mth-order impulse response estimate is given by

$$\tilde{\mathbf{h}}_{m_1,m_2} = \tilde{\mathbf{S}}_m\tilde{\mathbf{g}}_m^T, \quad \text{with } \tilde{\mathbf{S}}_m = \mathcal{H}_m(\mathbf{h}_M)\mathcal{T}_m^T(\mathbf{h}_M). \quad (9)$$

In the sequel, we provide a first-order upper bound for $\|\tilde{\mathbf{h}}_{m_1,m_2} - \mathbf{h}_{m_1,m_2}\|_2$.

A. First-Order Analysis

Result 1: If $\mathbf{h}_{m_1,m_2}$ is the truncated mth-order significant part of the true channel $\mathbf{h}_M$, with $\|\mathbf{h}_M\|_2 = 1$, and the size of the unmodeled tails is $\|\mathbf{d}_{m_1,m_2}^z\|_2 = \epsilon_m$, then the mth-order LP method furnishes $\tilde{\mathbf{h}}_{m_1,m_2}$ for which there holds to a first-order, with respect to ϵ_m

$$\left\|\tilde{\mathbf{h}}_{m_1,m_2} - \mathbf{h}_{m_1,m_2}\right\|_2 \le \frac{(2m+3)\epsilon_m}{\|\mathbf{h}_{(m_1)}\|_2}\sqrt{1 + \frac{1}{\delta_m^2}} + \frac{\sqrt{m+1}\epsilon_m}{\delta_m} \quad (10)$$

with $\delta_m \triangleq \sigma_{2m}(\mathcal{T}_{m-1}(\mathbf{h}_{m_1,m_2}))$, where $\sigma_i(\mathcal{M})$ denotes the ith singular value of matrix $\mathcal{M}$.

The proof is given in the Appendix.

Using arguments similar to those used in [5, Th. 2], it can be shown that

$$\delta_m \le \min\left(\|\mathbf{h}_{(m_1)}\|_2, \|\mathbf{h}_{(m_2)}\|_2\right). \quad (11)$$

From these two bounds, it becomes clear that if $\mathbf{h}_{(m_1)}$ and $\mathbf{h}_{(m_2)}$ are "large," i.e., $O(1)$, then the performance limitations of the mth-order LP method are dominated by δ_m. This can happen when m is chosen small enough so that the mth-order significant part of the true channel does not contain any "small" leading and/or trailing terms. If δ_m is sufficiently large, with respect to ϵ_m, then the mth-order LP method provides good channel approximations; otherwise, its performance may be poor. Since δ_m is the distance, in the matrix 2-norm, of $\mathcal{T}_{m-1}(\mathbf{h}_{m_1,m_2})$ from the matrices with rank one less than the assumed rank, it may be considered to be a measure of *diversity* of $\mathbf{h}_{m_1,m_2}$.

If, on the other hand, $\mathbf{h}_{(m_1)}$ and/or $\mathbf{h}_{(m_2)}$ are "small," i.e., $O(\epsilon_m)$, then $\delta_m = O(\epsilon_m)$, that is, δ_m becomes of the order of the perturbation, making the approximation problem *ill-conditioned* and leading to potentially poor performance of the mth-order LP method. This happens when m is greater than the *effective channel order* [8] and means that we try to model not only the significant part of the true channel but also some "small" leading and/or trailing impulse response terms. This case is termed *effective overmodeling*.

The sensitivity of the LP method, with respect to effective overmodeling, runs counter to recent claims of robustness of the LP method, with respect to overmodeling [3], [4]. The explanation is simple. In these works, overmodeling has been defined with respect to identically zero impulse response terms. As a result, the multichannel linear predictor is a solution to a rank-deficient system of linear equations. In [3], a solution has been computed by using pseudoinversion, whereas, in [4], by using order-recursions. However, a more natural definition of (effective) overmodeling is with respect to "small" impulse response terms [5], [8]. In this case, the linear predictor is the solution of a close to rank-deficient linear system [recall that in this case, $\delta_m = O(\epsilon_m)$]. In order to avoid large enhancement of inaccuracies, which are due to

$$\tilde{\mathbf{R}}_m = \begin{bmatrix} \tilde{\mathbf{r}}_0 & \tilde{\underline{\mathbf{r}}}_m \\ \tilde{\underline{\mathbf{r}}}_m^T & \tilde{\mathbf{R}}_{m-1} \end{bmatrix} = \mathcal{T}_m(\mathbf{h}_M)\mathcal{T}_m^T(\mathbf{h}_M) = \mathcal{T}_m(\mathbf{h}_{m_1,m_2}^z + \mathbf{d}_{m_1,m_2}^z)\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}^z + \mathbf{d}_{m_1,m_2}^z)$$
$$= \mathbf{R}_m + \underbrace{\left\{\mathcal{T}_m(\mathbf{h}_{m_1,m_2}^z)\mathcal{T}_m^T(\mathbf{d}_{m_1,m_2}^z) + \mathcal{T}_m(\mathbf{d}_{m_1,m_2}^z)\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}^z) + O(\epsilon_m^2)\right\}}_{\Delta\mathbf{R}_m}. \quad (6)$$

the influence of the tails, both the pseudoinversion and the order-recursions should be implemented with great care. Before the pseudoinversion, we should compute the effective rank of the data covariance matrix, whereas the order recursions should terminate at the right point by using an appropriate stopping criterion. These regularizing actions demand, explicitly or implicitly, the detection of the effective rank of the data covariance matrix, which is synomynous with effective channel order detection [8]. If we can estimate accurately the effective channel order, then we can implement these regularization techniques successfully. However, in this case, overmodeling seems superfluous. Consequently, the LP method, contrary to current beliefs, is *not* inherently robust with respect to effective channel overmodeling.

From Result 1, it becomes clear that if m is chosen favorably, the model quality depends on the size of the first "significant" term $\mathbf{h}_{(m_1)}$. Thus, in our study, term $\mathbf{h}_{m_1}$ plays the role that the first nonzero term $\mathbf{h}_{(0)}$ played in previous studies [3], [4], which assumed exact knowledge of the channel length and no tails.

The important problem of effective channel order detection is studied in [8].

One may ask how can we estimate the start and end points of the mth-order significant part m_1 and m_2. The answer is that their values are insignificant since during the equalization step, the fact that $m_1 \geq 0$ changes the solution by adding a delay of m_1 time units [5].

IV. Simulations

In the previous section, we derived bound (10), which provides significant insight into the performance of the LP method in realistic cases. This bound is derived by repeated application of the triangle and submultiplicative inequalities (see the Appendix). Thus, it is, in general, loose. However, it is given by a reasonably simple expression, identifying the cases in which the LP method performs well or may perform poorly. For example, it reveals the instability related to effective overmodeling.

In our simulation, we consider the mean asymptotic performance of the second-order LP method by varying the size of the tails. The significant part of the channel is

$$\mathbf{h}_{2,4} = [-0.6804\ 0.4281;\ 0.1777\ -0.2446;\ -0.0902\ -0.5043].$$

We construct $\mathbf{h}_{10} = \mathbf{h}^{z}_{2,4} + \mathbf{d}^{z}_{2,4}$ by using random "tails" $\mathbf{d}^{z}_{2,4}$ with

$$20 \leq 20\log_{10}\left(\|\mathbf{h}^{z}_{2,4}\|_2 / \|\mathbf{d}^{z}_{2,4}\|_2\right) \leq 60.$$

We scale $\mathbf{h}_{10}$ so that $\|\mathbf{h}_{10}\|_2 = 1$. Then, we apply the second-order LP method, and we compute $\tilde{\mathbf{h}}_{2,4}$. In Fig. 1, we plot the first-order bound (10) (thick line) and the actual error. We observe that although the bound is loose, it provides an indication of the quality of the estimator.

V. Conclusion

We considered the performance of the mth-order LP method for blind channel identification, when the true channel is of order M, with $m < M$. We showed that the mth-order LP method furnishes an approximation to the mth-order significant part of the true channel. The closeness depends on the diversity of the mth-order significant part and the size of the unmodeled tails. Furthermore, we showed that, contrary to current beliefs, the LP method is not inherently robust with respect to effective channel overmodeling.

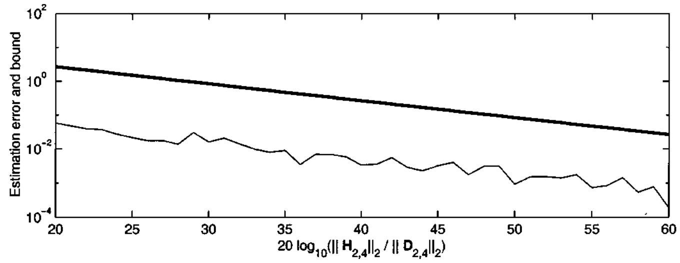

Fig. 1. Estimation error bound (10) (thick line) and measured error.

Appendix

Before proceeding to the proof of Result 1, we state the following Lemma.

Lemma: If $\mathbf{h}_{m_1,m_2}$ is the truncated mth-order significant part of the true channel, then the mth-order LP method furnishes $\tilde{\mathbf{h}}_{m_1,m_2}$, for which there holds, to a first-order approximation, with respect to the size of the unmodeled tails ϵ_m

$$\begin{aligned}\tilde{\mathbf{h}}_{m_1,m_2} - \mathbf{h}_{m_1,m_2} &= \Delta\mathbf{S}_m\mathbf{g}_m^T - \frac{1}{2}\mathbf{g}_m\Delta\mathbf{R}_m\mathbf{g}_m^T\mathbf{h}_{m_1,m_2} \\ &\quad - \mathbf{S}_m\begin{bmatrix}\mathbf{0} \\ \mathbf{R}_{m-1}^{-1}\left[\Delta\underline{\mathbf{r}}_m^T\ \Delta\mathbf{R}_{m-1}\right]\mathbf{g}_m^T\end{bmatrix}. \qquad (12)\end{aligned}$$

The proof can be derived by following steps analogous to those of ([3, App. A]). The error in quantity X is $\Delta X \triangleq \tilde{X} - X$, with $\tilde{X}$ defined in (6)–(9) and X defined in (1)–(4). Furthermore, instead of pseudoinversion, we use inversion.

Proof of Result 1: In order to prove Result 1, we will bound the 2-norm of each term of the right-hand side of (12). From (4) and (9), we deduce

$$\begin{aligned}\Delta\mathbf{S}_m \triangleq \tilde{\mathbf{S}}_m - \mathbf{S}_m &= \mathcal{H}_m(\mathbf{h}_M)\mathcal{T}_m^T(\mathbf{h}_M) \\ &\quad - \mathcal{H}_m(\mathbf{h}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}) \\ &= \mathcal{H}_m(\mathbf{h}^{z}_{m_1,m_2} + \mathbf{d}^{z}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}^{z}_{m_1,m_2} + \mathbf{d}^{z}_{m_1,m_2}) \\ &\quad - \mathcal{H}_m(\mathbf{h}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}) \\ &= \mathcal{H}_m(\mathbf{h}^{z}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{d}^{z}_{m_1,m_2}) \\ &\quad + \mathcal{H}_m(\mathbf{d}^{z}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}^{z}_{m_1,m_2}) + O(\epsilon_m^2)\end{aligned}$$

and thus

$$\begin{aligned}\Delta\mathbf{S}_m\mathbf{g}_m^T &= \mathcal{H}_m(\mathbf{h}^{z}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{d}^{z}_{m_1,m_2})\mathbf{g}_m^T \\ &\quad + \mathcal{H}_m(\mathbf{d}^{z}_{m_1,m_2})\mathbf{e}_{m_1} + O(\epsilon_m^2)\end{aligned}$$

where $\mathbf{e}_i$ is the canonical vector with 1 at the $(i+1)$st position and zeros elsewhere. It is easy to see that the second term vanishes. Using the matrix 2-norm/F-norm inequality, the structure of $\mathcal{H}_m(\mathbf{h}^{z}_{m_1,m_2})$ and $\mathcal{T}_m(\mathbf{d}^{z}_{m_1,m_2})$, and (5), we obtain the first-order bound

$$\begin{aligned}\left\|\Delta\mathbf{S}_m\mathbf{g}_m^T\right\|_2 &\leq \left\|\mathcal{H}_m(\mathbf{h}^{z}_{m_1,m_2})\right\|_F \left\|\mathcal{T}_m^T(\mathbf{d}^{z}_{m_1,m_2})\right\|_F \|\mathbf{g}_m\|_2 \\ &\leq \epsilon_m(m+1)\|\mathbf{g}_m\|_2. \qquad (13)\end{aligned}$$

For the second term, we have, from (6)

$$\begin{aligned}\mathbf{g}_m\Delta\mathbf{R}_m\mathbf{g}_m^T &= \mathbf{g}_m\Big\{\mathcal{T}_m(\mathbf{h}^{z}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{d}^{z}_{m_1,m_2}) \\ &\quad + \mathcal{T}_m(\mathbf{d}^{z}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}^{z}_{m_1,m_2}) + O(\epsilon_m^2)\Big\}\mathbf{g}_m^T \\ &= 2\mathbf{e}_{m_1}^T\mathcal{T}_m^T(\mathbf{d}^{z}_{m_1,m_2})\mathbf{g}_m^T + O(\epsilon_m^2) \\ &= 2\mathbf{g}_m\begin{bmatrix}\mathbf{0} \\ \mathbf{h}_{m_1-1} \\ \vdots \\ \mathbf{h}_{m_1-m}\end{bmatrix} + O(\epsilon_m^2)\end{aligned}$$

with $\mathbf{h}_k = \mathbf{0}$, for $k < 0$. Thus, to a first-order with respect to ϵ_m

$$\left\| \frac{1}{2}\mathbf{g}_m \Delta\mathbf{R}_m \mathbf{g}_m^T \mathbf{h}_{m_1,m_2} \right\|_2 \leq \left\| \frac{1}{2}\mathbf{g}_m \Delta\mathbf{R}_m \mathbf{g}_m^T \right\|_2 \leq \epsilon_m \left\| \mathbf{g}_m \right\|_2. \quad (14)$$

The third term of (12), which is denoted by T_3, can be written as

$$\begin{aligned} T_3 &= \mathcal{H}_m(\mathbf{h}_{m_1,m_2})\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}) \\ &\quad \times \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathcal{T}_{m-1}^{-T}(\mathbf{h}_{m_1,m_2})\mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1,m_2}) \end{bmatrix} \\ &\quad \times \Big\{ \mathcal{T}_m(\mathbf{h}_{m_1,m_2}^z)\mathcal{T}_m^T(\mathbf{d}_{m_1,m_2}^z) \\ &\quad + \mathcal{T}_m(\mathbf{d}_{m_1,m_2}^z)\mathcal{T}_m^T(\mathbf{h}_{m_1,m_2}^z) + O(\epsilon_m^2) \Big\} \mathbf{g}_m^T \\ &= \mathcal{H}_m(\mathbf{h}_{m_1,m_2}) \left\{ \begin{bmatrix} \mathbf{0}^\mathbf{T} & \mathbf{0}^\mathbf{T} & \mathbf{0}^\mathbf{T} \\ \mathbf{O} & \mathbf{I}_{2m} & \mathbf{O} \end{bmatrix} \mathcal{T}_m^T(\mathbf{d}_{m_1,m_2}^z)\mathbf{g}_m^T \right. \\ &\quad \left. + \begin{bmatrix} \mathbf{0} \\ \mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1,m_2}) \begin{bmatrix} \mathbf{h}_{m_1-1} \\ \vdots \\ \mathbf{h}_{m_1-m} \end{bmatrix} \end{bmatrix} \right\}. \end{aligned}$$

Finally

$$\begin{aligned} \|T_3\|_2 &\leq \|\mathcal{H}_m(\mathbf{h}_{m_1,m_2})\|_F \left\{ \left\|\mathcal{T}_m(\mathbf{d}_{m_1,m_2}^z)\right\|_F \|\mathbf{g}_m\|_2 \right. \\ &\quad \left. + \left\|\mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1,m_2})\right\|_2 \left\| \begin{bmatrix} \mathbf{h}_{m_1-1} \\ \vdots \\ \mathbf{h}_{m_1-m} \end{bmatrix} \right\|_2 \right\} \\ &\leq (m+1)\epsilon_m \|\mathbf{g}_m\|_2 + \frac{\sqrt{m+1}\epsilon_m}{\delta_m}. \end{aligned} \quad (15)$$

In order to bound the term $\|\mathbf{g}_m\|_2$ appearing in (13)–(15), we proceed as follows. Since

$$\begin{aligned} &\mathcal{A}_m \left\{ \mathcal{T}_{m-1}(\mathbf{h}_{m_1,m_2})\mathcal{T}_{m-1}^T(\mathbf{h}_{m_1,m_2}) \right\} \\ &\quad = -[\mathbf{h}_{(m_1+1)} \quad \cdots \quad \mathbf{h}_{(m_2)} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}]\mathcal{T}_{m-1}^T(\mathbf{h}_{m_1,m_2}) \end{aligned}$$

and $\mathcal{T}_{m-1}(\mathbf{h}_{m_1,m_2})$ is nonsingular, we obtain

$$\begin{aligned} \|\mathcal{A}_m\|_2 &\leq \left\|[\mathbf{h}_{(m_1+1)} \quad \cdots \quad \mathbf{h}_{(m_2)}]\right\|_F \left\|\mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1,m_2})\right\|_2 \\ &\leq \frac{1}{\delta_m}. \end{aligned}$$

Using the expression for the equalizer $\mathbf{g}_m = (1/\|\mathbf{h}_{(m_1)}\|_2)\mathbf{v}^T[\mathbf{I}\ \mathcal{A}_m]$ and the fact that $\|\mathbf{v}\|_2 = 1$, we obtain

$$\begin{aligned} \|\mathbf{g}_m\|_2 &\leq \frac{1}{\|\mathbf{h}_{(m_1)}\|_2} \|[\mathbf{I}\ \mathcal{A}_m]\|_2 = \frac{1}{\|\mathbf{h}_{(m_1)}\|_2}\sqrt{1+\|\mathcal{A}_m\|_2^2} \\ &\leq \frac{1}{\|\mathbf{h}_{(m_1)}\|_2}\sqrt{1+\frac{1}{\delta_m^2}}. \end{aligned} \quad (16)$$

Putting (16) in (13)–(15) and adding the derived bounds, we prove Result 1.

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Interference Blanking Probabilities for SLB in Correlated Gaussian Clutter Plus Noise

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***Abstract*—This work presents closed-form expressions of the probability P_{FA} of false alarm, the probability P_{TB} of blanking a target received in the main lobe, and the probability P_B of blanking a coherent repeater interference (CRI) via an SLB device operating in correlated Gaussian clutter with known Doppler spectrum plus white Gaussian thermal noise.**

***Index Terms*—Coherent interference rejection, radar clutter, sidelobe blanking.**

I. Introduction and Working Principle

To counter impulsive-type sidelobe interference, a radar receiver usually employs a sidelobe blanking (SLB) system, which blanks the radar receiver output in those range cells where the unwanted signal appears in the radar antenna sidelobes. The purpose is reached via a low gain auxiliary antenna that is located close to the main antenna, which receives the same interfering signals of the main antenna. By comparing the signals captured by the radar and auxiliary antennas, we may ascertain whether the impulsive signal is received through the radar antenna side lobes. In such a situation, the radar signal is blanked in the range cell affected by the interference. The processing scheme connecting the radar and the SLB device is outlined in Fig. 1 [1], [2]. The SLB decides whether or not to blank the main channel

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